MIDTERM 1 (VOJTA) - ANSWER KEY

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(1) (a) $\begin{bmatrix}1\\5\end{bmatrix}$

(b) $\begin{bmatrix}2\\3\end{bmatrix}$

(calculate A, A^2, A^3 and notice the pattern! In particular $A^n = I$ if n is even)

(c) Undefined (dimensions don't agree)

(d) $\begin{bmatrix}-23 & -2\\3 & 9\end{bmatrix}$

(2)

$$\begin{bmatrix}x\\y\\z\\u\\v\end{bmatrix} = \begin{bmatrix}3\\-\frac{1}{2}\\0\\\frac{1}{2}\\2\end{bmatrix} + z\begin{bmatrix}\frac{7}{2}\\-\frac{3}{2}\\1\\0\\0\end{bmatrix}$$

(z is free)

Another way of writing this is:

$$\begin{cases} x = 3 + \frac{7}{2}z \\ y = -\frac{1}{2} - \frac{3}{2}z \\ z = z \\ u = \frac{1}{2} \\ v = 2 \end{cases}$$

(3) $A^{-1} = \begin{bmatrix}-1 & -2 & -1\\-1 & 3 & 1\\-1 & \frac{3}{2} & \frac{1}{2}\end{bmatrix}$

(4) An $n \times n$ matrix A is **invertible** if there exists a matrix B such that $AB = BA = I$, where I is the $n \times n$ identity matrix.

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Three conditions that are equivalent to invertibility are (choose your 3 favorite ones):

(a) A is row-equivalent to the $n \times n$ identity matrix

(b) A has n pivot positions

(c) $A\mathbf{x} = \mathbf{0}$ has only the trivial solution

(d) $A\mathbf{x} = \mathbf{b}$ has a solution for every $\mathbf{b}$ in $\mathbb{R}^n$

(e) The columns of A form a linearly independent set

(f) The columns of A span $\mathbb{R}^n$

(g) The associated linear transformation is one-to-one

(h) The associated linear transformation is onto

(5) Ignore this

(6) No, because it is not closed under scalar multiplication! For example, $\mathbf{v} = 1$ is in $\mathbb{Z}$, but $\sqrt{2}\mathbf{v} = \sqrt{2}$ is not in $\mathbb{Z}$.

(7) Yes! For example, form the matrix:

$$A = \begin{bmatrix} 1 & 2 & 7 \\ 0 & 4 & 3 \\ 0 & 7 & 5 \end{bmatrix}$$

And solve $A\mathbf{x} = \mathbf{0}$.
You should get $\mathbf{x} = \mathbf{0}$