MATH 3A – MIDTERM STUDY GUIDE

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GENERAL INFO

The Midterm Exam takes place on Wednesday, February 13, during the usual time (1-1:50 pm or 2-2:50 pm) and the usual place (SE2 1304). Please make sure to take the exam in the lecture you're officially enrolled in, not the one you prefer to go to. Also bring your student ID, as we'll be checking IDs during the exam. There will be a seating chart for the exam, which I'll send out a day or two before the exam.

This is the study guide for the exam. Please read it carefully, because it contains a lot of info about what's going to be on the exam and what I expect you to know or not to know. That said, remember that this study guide is just a *guide* and not a complete list. I've tried to make this list as complete as possible, but there are always things that I may have missed.

The midterm covers sections 1.1-1.5, 1.7-1.9, 2.1-2.3, 2.8-2.9, and 3.1-3.3.

There will be 7 problems on the exam. Problem 1 will be 10 True/False questions where you *don't* have to justify your answer, Problems 2-5 will be computational questions, Problem 6 will have True/False questions where you have to justify your answer, meaning: If the answer is **True**, you have to explain why it's true, possibly citing a theorem covered in class (e.g. the Invertible Matrix Theorem). If the answer is **False**, you have to give an *explicit* counterexample, and show why that example works. Finally, Problems 7 is trickier than the rest.

Beware: In lecture, I tried to hit the main points of each section, but there are topics in the book and/or the suggested homework that I didn't cover in lecture but that I expect you to know for the exam. That's why it's especially important that you look at this study guide, in order to avoid surprises. The *best* way to prepare for the exam is to (1) review your notes from lecture, (2) look at the suggested homework, (3) look at the YouTube videos I posted (except the ones that are labeled optional), (4) read the sections in the book,

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and (5) do the practice exams on my website. You don't need to study the quizzes or the discussion worksheets, since I didn't look at them at all.

Warning: The practice exams on my website are a bit easier than the exam will be (for instance, the average of the first practice exam was a 95/100, which is a bit too high), so don't *just* study the practice exams. Also look at this study guide and the suggested problems. If you want some challenging problems, the old quizzes on my website have some trickier problems!

Finally, remember that I'm on your side; if the average of this exam and on the final is lower than I expected, then I will curve the class, in the sense that I will take all your raw scores, add them up, and then curve that total raw score. I won't curve individual exams. You can find the official grading percentages on the syllabus.

Note: 1.1.11 means Problem 11 in section 1.1. Remember that there is a hint/solution-bank on my website as well.

YouTube Playlists

There are a lot of videos on my YouTube channel, based on the concepts covered in this course. Check them out if you need help with a topic:

- Chapter 1: Linear Equations in Linear Algebra
- Chapter 2: Matrix Algebra
- Chapter 3: Determinants

Chapter 1: Linear Equations in Linear Algebra

- Solve a system of equations, or determine if there are no solutions. Try to write your solutions in vector form. (1.1.11, 1.1.13, 1.2.11, 1.2.13, 1.4.11, 1.5.9, 1.5.12, Gaussian Elimination, No solutions, Infinitely many Solutions)
- Find values of h for which a system has a solution (1.1.21, 1.1.28)
- Know the existence/uniqueness theorem (Theorem 2 in section 1.2), it's a great way of checking if a system has a solution or not. See in particular 1.2.24 and 1.2.25.
- Determine if a given vector $\mathbf{b}$ is a linear combination of other vectors (1.3.11, 1.3.13, 1.3.15, 1.3.17, Span)
- In section 1.3, ignore the part on 'Linear Combinations in Applications'
- Also look at 1.3.25 and 1.3.26, people find that tricky.
- Know Theorem 6 in section 1.5; it's a very useful way of thinking of solutions of systems; in theory know how to prove this, see 1.5.25.

- Determine whether a set of vectors is linearly dependent or independent (1.7.5, 1.7.7, 1.7.11, 1.7.15, 1.7.17, Linear Independence)
- You don't need to know Theorem 7 in 1.7 if it's too confusing to you, but know Theorems 8 and 9, they're useful.
- Show that T is or is not a linear transformation (1.8.32, 1.8.32, Linear Transformations).
- I could ask you to show that, for any matrix A, $T(\mathbf{x}) = A\mathbf{x}$ is a linear transformation.
- Ignore example 6 in 1.8
- Given a linear transformation T and a vector $\mathbf{b}$, determine whether $\mathbf{b}$ is in the image of T (1.8.3, 1.8.9)
- You don't need to know how to derive the formula for the rotation matrix (Example 3 in 1.9), but you need to memorize it (check out Rotation Matrix if you want)
- Do **NOT** memorize Tables 1-4 in 1.9
- Find the matrix of a given linear transformation T (1.9.1, 1.9.3, 1.9.5, 1.9.9, 1.9.11, 1.9.17)
- Determine if a linear transformation is one-to-one or onto (1.9.25, One-to-one, onto, matrix)
- Know the Row Theorem from lecture, and remember that rows, $A\mathbf{x} = \mathbf{b}$, span, and onto go together; check out Row Theorem and ax + by = e
- Know the Column Theorem from lecture, and remember that columns, $A\mathbf{x} = \mathbf{0}$, linear independence, and one-to-one go together.

CHAPTER 2: MATRIX ALGEBRA

- Given A and B, find AB, or say 'it does not exist' (2.1.5, 2.1.6, 2.1.9), Matrix Multiplication, AB vs. BA
- Calculate things like A^T, A^2 etc.
- Understand what AB means in terms of linear transformations
- Mnemonic: **In**put, **M**outhput, so $\mathbb{R}^n$ to $\mathbb{R}^m$
- You don't need to understand example 6 in section 2.1
- Remember that $(AB)^T = B^T A^T$ (reverse order)
- Find the inverse of a matrix A, including the formula in the 2×2 case, and the general procedure where you form the big matrix $[A|I]$ and row-reduce (2.2.1, 2.2.3, 2.2.31, 2.3.3, 2.3.7, Calculating A^{-1})
- Use A^{-1} to solve $A\mathbf{x} = \mathbf{b}$ (2.2.5)
- Remember that $(AB)^{-1} = B^{-1}A^{-1}$ (reverse order)
- Understand what A^{-1} means in terms of linear transformations.
- Ignore example 3 in 2.2

- Know the three types of elementary matrices and their inverses, but you **don't** need to know how to write a matrix in terms of elementary matrices, and you don't need to know the proof of Theorem 7.
- Ignore the section on 'Another View of Matrix Inversion'
- Understand all the implications of the invertible matrix theorem (IMT). You don't need to memorize them (I won't ask you 'List all the conditions of the IMT'), but know how to recognize it. For example, I could ask you "If A is $n \times n$ and the columns of A span $\mathbb{R}^n \cdots$," then in your mind you should be like "Ah! The IMT!" (2.3.15−29)
- Figure out when a matrix or a linear transformation is invertible (2.3.5, 2.3.33, A invertible)
- Always remember that the IMT says 'An invertible matrix is awesome,' so what you want to be true for an invertible matrix is true for an invertible matrix
- **Unless otherwise specified, do NOT assume A is square!!!** This is a main source of pitfalls!
- You don't need to know how to prove the IMT.
- Show H is or is not a subspace of $\mathbb{R}^n$ (2.8.1, 2.8.2, Subspace, Not a subspace)
- I could ask you to show that $Span\{\mathbf{u}, \mathbf{v}\}$ is a subspace, see Span is a subspace
- I could ask you to show that $Nul(A)$ is a subspace of $\mathbb{R}^n$ (Theorem 12 in 2.8), see Nul(A) is a subspace or $Col(A)$ is a subspace of $\mathbb{R}^m$ (simply because it's a span)
- Given A, find a basis for $Nul(A)$, find a basis for $Col(A)$, find $rank(A)$ and find $dim(Nul(A))$ (2.8.25, 2.8.26, 2.9.9, 2.9.12, Nul(A), Nul(A), Col(A), Rank(A)).
- Remember that for $Col(A)$, you need to go back to the columns of A, and for $Nul(A)$, you need the RREF.
- You can ignore any problems with $Row(A)$.
- Check if something is a basis or not (2.8.17, 2.8.20, Basis check)
- Given $\mathbf{x}$, find $[\mathbf{x}]_{\mathcal{B}}$ and vice-versa (2.9.1, 2.9.3, Coordinates)
- Find a basis and the dimension of the span of vectors (2.9.13, Basis and dimension) (it all boils down to finding a basis for the column space)
- Do problems involving the rank-nullity theorem (2.8.19-25)
- Intuitively, $Nul(A)$ measures how bad a matrix is, and $Col(A)$ or $rank(A)$ measures how good a matrix is. The rank-nullity theorem says that their sum balances out, like a conservation of energy.

- Don't worry about the Basis Theorem (Theorem 15 in 2.9, also known as the Dimension Theorem in lecture), although it's kind of useful
- For the invertible matrix theorem (section 2.9), again you don't *have* to memorize it, but understand what it's saying

CHAPTER 3: DETERMINANTS

- Calculate the determinant of a matrix, possibly using row-reductions (3.1.31, 3.1.3, 3.1.9, 3.1.13, 3.2.5, 3.2.7, 3.2.13, Determinants, Determinants and row-reduction, Another determinant)
- Use determinants to find if a matrix is invertible (3.2.23, 3.2.24, Determinants and Invertibility)
- In section 3.2, ignore the paragraph preceding Theorem 4, as well as the sections on 'A Linearity Property of the Determinant Function' and 'Proofs of Theorems 3 and 6'
- Solve questions using the fact that $\det(AB) = \det(A)\det(B)$ and $\det(A^T) = \det(A)$ and $\det(A^{-1}) = \frac{1}{\det(A)}$ (3.2.31−35)
- Solve a system using Cramer's rule (3.3.1, 3.3.3, 3.3.5, Cramer's Rule)
- Ignore the section 'Application to Engineering'
- Find inverses using determinants (3.2.13, 3.2.18. A-1 using determinants)
- In the sections on 'Determinants as Area or Volume' and 'Linear Transformations,' ignore the lengthy descriptions. You just need to know how to find areas of parallelograms (Example 4) and parallelipipeds, as well as the formula in Theorem 10 and Example 5 (3.3.19, 3.3.23, 3.3.27)
- Calculate volumes using determinants (3.3.29, 3.3.20, 3.3.31, 3.3.32, Determinants and Volumes)
- I looooooooooove determinants :)

TRUE/FALSE EXTRAVAGANZA

Do the following set of T/F questions: 1.1.23, 1.1.24, 1.2.21, 1.2.22, 1.3.23, 1.3.24, 1.4.23, 1.4.24, 1.5.23, 1.5.24*, 1.7.21*, 1.7.22, 1.8.21*, 1.8.22, 1.9.23, 1.9.24*, 2.1.15, 2.1.16, 2.2.9, 2.2.10, 2.3.11, 2.3.12, 2.8.21*, 2.8.22, 2.9.17, 2.9.18, 3.1.39, 3.1.40, 3.2.27*, 3.2.28 (The ones with * next to them have solutions in the Homework-Hints)

CONCEPTS

Understand the following concepts. In theory, also know the definitions of the concepts with *

- Pivots
- Row-echelon and reduced row-echelon form
- Span*
- Linear independence*
- Linear Transformation*
- One-to-one* and onto*
- Invertible matrix*
- Invertible Matrix Theorem
- Subspace*
- $Nul(A)$*, $Col(A)$*
- Dimension*
- Rank*
- Rank Theorem*
- Coordinates of $\mathbf{x}$ with respect to $\mathcal{B}$*
- Determinant