MATH 3A – MIDTERM

Name: ________________________

Student ID: ________________________

Instructions: Welcome to your Midterm! You have 50 minutes to take this exam, for a total of 100 points. No books, notes, calculators, or cellphones are allowed. Remember that you are not only graded on your final answer, but also on your work. If you need to continue your work on the back of the page, clearly indicate so, or else your work will be discarded. May your luck be infinite-dimensional! :)

Academic Honesty Statement: I hereby certify that the exam was taken by the person named and without any form of assistance and acknowledge that any form of cheating (no matter how small) results in an automatic F in the course, and will be further subject to disciplinary consequences, pursuant to section 102.1 of the UCI Student Code of Conduct.

Signature: __________________________________

1		20
2		15
3		10
4		10
5		20
6		10
7		15
Total		100

Date: Wednesday, February 13, 2019.

1. *(20 points, 2 points each)* Label each statement as **TRUE** or **FALSE**. In this question, you do **NOT** have to justify your answer. Each correct answer will get 2 points and each incorrect or illegible answer will get 0 points.

   (a) If A is a 4×6 matrix with 2 pivot columns, then $Nul(A) = \mathbb{R}^4$.

   (b) If $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$, then the columns of A must span $\mathbb{R}^m$

   (c) If $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a one-to-one linear transformation, then T must also be onto $\mathbb{R}^3$

   (d) A matrix A such that $\det(A^2) - 2\det(A) + \det(I) = 0$ must be invertible.

   (e) For any $\mathbf{b}$ in $\mathbb{R}^m$, the set of solutions to $A\mathbf{x} = \mathbf{b}$ is a subspace of $\mathbb{R}^n$.

   (f) If A does not have a pivot in every row, then $A\mathbf{x} = \mathbf{b}$ must be inconsistent for every $\mathbf{b}$.

   (g) If A and B are row-equivalent, then $Col(A) = Col(B)$

   (h) The matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ is not invertible.

   (i) If $AB = I$, then A must be invertible

   (j) If $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, then $Nul(A) = Span \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$

2. *(15 points, 5 points each)* In this question, you do **NOT** need to justify your answers, but there will be no partial credit for each subpart.

   Find all the values of h and k (if any) for which $A\mathbf{x} = \mathbf{b}$ has

   (a) No solutions
   (b) Exactly one solution
   (c) Infinitely many solutions

$$A = \begin{bmatrix} 1 & -2 \\ 2 & h \\ 3 & -6 \\ -1 & 2 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 3 \\ 6 \\ k \\ -3 \end{bmatrix}$$

3. *(10 points)* Calculate the following determinant.

$$\begin{vmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 4 & 5 & 0 & 8 & 0 \\ 3 & 9 & 0 & 1 & 0 & 0 & 1 \\ 4 & 5 & 7 & 0 & 3 & 5 & 7 \\ 8 & 3 & 0 & 1 & 0 & 1 & 2 \\ 3 & 2 & 0 & 1 & 0 & 2 & 1 \end{vmatrix}$$

4. *(10 points)* Solve the system $A\mathbf{x} = \mathbf{b}$

$$A = \begin{bmatrix} 2 & 1 & -3 \\ 1 & 2 & 4 \\ -4 & 4 & 0 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 6 \\ -1 \\ 0 \end{bmatrix}$$

5. *(20 = 10 + 1 + 4 + 2 + 3 points)* For the following matrix A, find:

(a) A basis for $Nul(A)$
(b) $\dim(Nul(A))$
(c) A basis for $Col(A)$
(d) $Rank(A)$
(e) State the Rank Theorem

$$A = \begin{bmatrix} 1 & 1 & -3 & 7 & 9 \\ 1 & 2 & -4 & 10 & 13 \\ 1 & -1 & -1 & 1 & 1 \\ 1 & -3 & 1 & -5 & -7 \\ 1 & -2 & 0 & 0 & -5 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & -3 & 7 & 9 \\ 0 & 1 & -1 & 3 & 4 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

6. *(10 points, 5 points each)* Label each statement as **TRUE** or **FALSE**. In this question, you **HAVE** to justify your answer, meaning that if the statement is true, you have to explain why it's true, and if the statement is false, you have to give an explicit counterexample and show why it's a counterexample.

   (a) The following set is a subspace of $\mathbb{R}^2$:

$$H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \text{ in } \mathbb{R}^2 \text{ such that } x^2 + y^2 \leq 1 \right\}$$

   (b) The linear transformation T from $\mathbb{R}^2$ to $\mathbb{R}^2$ which reflects points in the plane about the $y-$axis is onto $\mathbb{R}^2$.

7. *(15 points, 5 points each)* Let R be the triangle in $\mathbb{R}^2$ with vertices $(0,0),(1,0),(0,1)$ and R' be the triangle in $\mathbb{R}^2$ with vertices $(a,b),(c,d),(e,f)$.

(a) Find the matrix A of the linear transformation T that sends R to R', that is such that $T(R) = R'$.

(b) Using the formula of determinants in terms of areas, calculate the area of R'. No need to foil out the expression you found.

(c) Show that your answer in (b) can be written as (the absolute value of)

$$\frac{1}{2}\begin{vmatrix} 1 & a & b \\ 1 & c & d \\ 1 & e & f \end{vmatrix}$$