MATH 54 – QUIZ 1 – SOLUTIONS

PEYAM RYAN TABRIZIAN

1. *(5 points)* Using the **reduced row-echelon form**, find the general solution of the linear system corresponding to the following **augmented** matrix.

$$\begin{bmatrix} 2 & -3 & 5 & 5 \\ 1 & -2 & -1 & -5 \\ 3 & -4 & 11 & 15 \end{bmatrix}$$

First interchange the first and second rows (it's better to have 1's on top):

$$\begin{bmatrix} 1 & -2 & -1 & -5 \\ 2 & -3 & 5 & 5 \\ 3 & -4 & 11 & 15 \end{bmatrix}$$

Now subtract 2 times the first row from the second, and 3 times the first row from the third:

$$\begin{bmatrix} 1 & -2 & -1 & -5 \\ 0 & 1 & 7 & 15 \\ 0 & 2 & 14 & 30 \end{bmatrix}$$

Now divide the third row by 2:

$$\begin{bmatrix} 1 & -2 & -1 & -5 \\ 0 & 1 & 7 & 15 \\ 0 & 1 & 7 & 15 \end{bmatrix}$$

And subtract the second row from the third:

$$\begin{bmatrix} 1 & -2 & -1 & -5 \\ 0 & 1 & 7 & 15 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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This is in row-echelon form, but we want the reduced row-echelon form. For this, add 2 times the second row to the first:

$$\begin{bmatrix} 1 & 0 & 13 & 25 \\ 0 & 1 & 7 & 15 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

which gives

$$\begin{cases} x_1 + 13x_3 = 25 \\ x_2 + 7x_3 = 15 \\ x_3 = x_3 \end{cases} \Rightarrow \begin{cases} x_1 = 25 - 13x_3 \\ x_2 = 15 - 7x_3 \\ x_3 = x_3 \end{cases} = \begin{bmatrix} 25 \\ 15 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -13 \\ -7 \\ 1 \end{bmatrix}$$

where x_3 is free.

2. *(5 points)* For what c is $\begin{bmatrix} c \\ 0 \\ c \end{bmatrix}$ in Span $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} c \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \right\}$?

In other words, we'd like to see for which c there are x_1, x_2, x_3, x_4 such that:

$$x_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} c \\ 1 \\ 1 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} c \\ 0 \\ -c \end{bmatrix}$$

In other words, the question is: for which c is the following system consistent:

$$\begin{cases} x_1 + x_2 + cx_3 + x_4 & = c \\ -x_2 + x_3 + 2x_4 & = 0 \\ x_1 + 2x_2 + x_3 - x_4 & = -c \end{cases}$$

Let's find the row-echelon form of the augmented matrix:

$$\begin{bmatrix} 1 & 1 & c & 1 & c \\ 0 & -1 & 1 & 2 & 0 \\ 1 & 2 & 1 & -1 & -c \end{bmatrix}$$

Substracting the first row from the third, we get:

$$\begin{bmatrix} 1 & 1 & c & 1 & c \\ 0 & -1 & 1 & 2 & 0 \\ 0 & 1 & 1-c & -2 & -2c \end{bmatrix}$$

Adding the second row to the third, we get:

$$\begin{bmatrix} 1 & 1 & c & 1 & c \\ 0 & -1 & 1 & 2 & 0 \\ 0 & 0 & 2-c & 0 & -2c \end{bmatrix}$$

By the Fact discussed in section, this system has a solution if and only if $2 - c \neq 0$ (otherwise we'd have a row of the form $\begin{bmatrix} 0 & 0 & 0 & 0 & \star \end{bmatrix}$, where $\star \neq 0$), which gives $\boxed{c \neq 2}$