MATH 54 – QUIZ 5 – SOLUTIONS

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1. *(5 points)* For the following matrix A, find:

$$A = \begin{bmatrix} 1 & 1 & -3 & 7 & 9 & -9 \\ 1 & 2 & -4 & 10 & 13 & -12 \\ 1 & -1 & -1 & 1 & 1 & -3 \\ 1 & -3 & 1 & -5 & -7 & 3 \\ 1 & -2 & 0 & 0 & -5 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & -3 & 7 & 9 & -9 \\ 0 & 1 & -1 & 3 & 4 & -3 \\ 0 & 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) A basis for $Row(A)$

The pivots are in the first three rows of A, hence a basis for $Row(A)$ is:

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ -3 \\ 7 \\ 9 \\ -9 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 3 \\ 4 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ -2 \end{bmatrix} \right\}$$

(b) A basis for $Col(A)$

The pivots are in the first, second, and fourth columns of A, hence a basis for $Col(A)$ is (remember to go back to the **original** columns of A):

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \\ -3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ 10 \\ 1 \\ -5 \\ 0 \end{bmatrix} \right\}$$

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(c) $Rank(A)$

$Rank(A) =$ number of pivots of $A = 3$

(d) $\dim Nul(A)$

By the rank-theorem, $rank(A) + \dim Nul(A) = 6$ (number of columns of A), hence $\dim Nul(A) = 6 - rank(A) = 6 - 3 = 3$

2. *(5 points)* Let $\mathcal{A} = \left\{ \begin{bmatrix} -1 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ -7 \end{bmatrix} \right\}$ and $\mathcal{D} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$.

Use a **change-of-coordinates matrix** to find $[\mathbf{x}]_{\mathcal{A}}$ given

$[\mathbf{x}]_{\mathcal{D}} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

We have:

$$[\mathbf{x}]_{\mathcal{A}} = P_{\mathcal{A} \leftarrow \mathcal{D}} [\mathbf{x}]_{\mathcal{D}}$$

Now to find $P_{\mathcal{A} \leftarrow \mathcal{D}}$, row-reduce:

$$[\mathcal{A} \mid \mathcal{D}] = \begin{bmatrix} -1 & 1 & 1 & 1 \\ 8 & -7 & 2 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 9 & 8 \\ 0 & 1 & 10 & 9 \end{bmatrix} = [I \mid P_{\mathcal{A} \leftarrow \mathcal{D}}]$$

Therefore:

$$P_{\mathcal{A} \leftarrow \mathcal{D}} = \begin{bmatrix} 9 & 8 \\ 10 & 9 \end{bmatrix}$$

And hence:

$$[\mathbf{x}]_{\mathcal{A}} = P_{\mathcal{A} \leftarrow \mathcal{D}} [\mathbf{x}]_{\mathcal{D}} = \begin{bmatrix} 9 & 8 \\ 10 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} -15 \\ -17 \end{bmatrix}$$