MATH 54 − QUIZ 6

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Instructions: You have 20 minutes to take this quiz, for a total of 10 points. May your luck be diagonalizable!

1. *(6 points)*
   (a) *(4 points)* Find a diagonal matrix D and an invertible matrix P such that $A = PDP^{-1}$, where:

$$A = \begin{bmatrix} 1 & 1 & -1 \\ -3 & -3 & 1 \\ -3 & -3 & 1 \end{bmatrix}$$

Note: In order to save you some time, you may assume that an eigenvector of A corresponding to $\lambda = 0$ is $\mathbf{v_2} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ and an eigenvector corresponding to $\lambda = 1$ is $\mathbf{v_3} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

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(b) *(2 points)* Suppose $\mathbf{v} = \mathbf{v_1} + \mathbf{v_2} - \mathbf{v_3}$, where $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ are the three eigenvectors you found above. Find $A^{2015}\mathbf{v}$

2. *(4 points)* Define $T : M_{2\times 2} \to M_{2\times 2}$ by:

$$T(A) = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} A$$

Find the matrix of T relative to the basis:

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\} \text{ of } M_{2\times 2}$$