MATH 54 – QUIZ 7 – SOLUTIONS

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1. *(7 points)* Let $W = Span\{\mathbf{u}, \mathbf{v}\}$, where $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ -2 \\ 2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 4 \end{bmatrix}$.

Find the orthogonal projection of $\mathbf{x} = \begin{bmatrix} 3 \\ -1 \\ 2 \\ 1 \end{bmatrix}$ on W.

CAREFUL!!! The set $\{\mathbf{u}, \mathbf{v}\}$ is **NOT** orthogonal, so you **CANNOT** use the projection-formula directly! First, apply Gram-Schmidt to $\{\mathbf{u}, \mathbf{v}\}$ to get an orthogonal basis $\{\mathbf{u}', \mathbf{v}'\}$ of W:

$$\mathbf{u}' = \mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ -2 \\ 2 \end{bmatrix}$$

$$\hat{\mathbf{v}} = \left(\frac{\mathbf{v} \cdot \mathbf{u}'}{\mathbf{u}' \cdot \mathbf{u}'}\right) \mathbf{u}' = \left(\frac{9}{9}\right) \begin{bmatrix} 1 \\ 0 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -2 \\ 2 \end{bmatrix}$$

$$\mathbf{v}' = \mathbf{v} - \mathbf{u}' = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 4 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 2 \\ 2 \end{bmatrix}$$

NOW you can apply the orthogonal projection formula of $\mathbf{x}$ to $W = Span\{\mathbf{u}', \mathbf{v}'\}$:

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$$\hat{\mathbf{x}} = \left(\frac{\mathbf{x}\cdot\mathbf{u}'}{\mathbf{u}'\cdot\mathbf{u}'}\right)\mathbf{u}' + \left(\frac{\mathbf{x}\cdot\mathbf{v}'}{\mathbf{v}'\cdot\mathbf{v}'}\right)\mathbf{v}' = \left(\frac{1}{9}\right)\begin{bmatrix}1\\0\\-2\\2\end{bmatrix} + \left(\frac{7}{9}\right)\begin{bmatrix}0\\-1\\2\\2\end{bmatrix} = \begin{bmatrix}1/9\\-7/9\\4/3\\16/9\end{bmatrix}$$

2. *(3 points)* Given a vector $\mathbf{x}$ and a subspace W, find a formula for the orthogonal projection of $\mathbf{x}$ on $W^{\perp}$.

Hint: A picture might help!

The answer is $\boxed{\mathbf{x}-\hat{\mathbf{x}}}$ (where $\hat{\mathbf{x}}$ is the orthogonal projection of $\mathbf{x}$ on W), as illustrated in the following picture:

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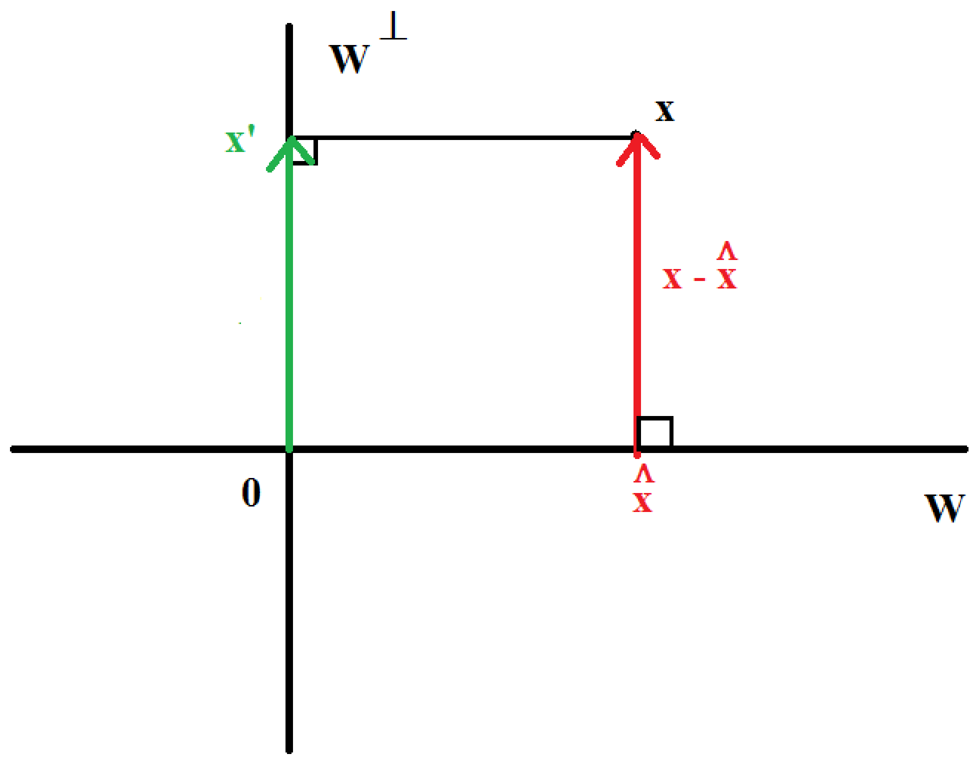

In this picture, $\mathbf{x}'$ is the orthogonal projection of $\mathbf{x}$ on $W^{\perp}$. Notice that the green vector equals to the red vector, so $\mathbf{x}' = \mathbf{x} - \hat{\mathbf{x}}$