MATH 54 – TRUE/FALSE QUESTIONS FOR MIDTERM 1

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1. (a) If the **augmented** matrix of the system $A\mathbf{x} = \mathbf{b}$ has a pivot in the last column, then the system $A\mathbf{x} = \mathbf{b}$ has no solution.

   (b) If A and B are invertible 2×2 matrices, then $(AB)^{-1} = A^{-1}B^{-1}$

   (c) If A is a 3×3 matrix such that the system $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$ in $\mathbb{R}^3$.

   (d) The general solution to $A\mathbf{x} = \mathbf{b}$ is of the form $\mathbf{x} = \mathbf{x}_p + \mathbf{x}_0$, where $\mathbf{x}_p$ is a *particular* solution to $A\mathbf{x} = \mathbf{b}$ and $\mathbf{x}_0$ is the *general* solution to $A\mathbf{x} = \mathbf{0}$.

   (e) If P and D are $n \times n$ matrices, then $det(PDP^{-1}) = det(D)$

   (f) If $T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix}$, then $Nul(T) = \text{Span}\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$

   (g) The set of polynomials $\mathbf{p}$ in P_2 such that $\mathbf{p}(3) = 0$ is a subspace of P_2

   (h) $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$

2. (a) If A and B are any 2×2 matrices, then $AB = BA$

   (b) The matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 0 & 3 & 0 \end{bmatrix}$ is not invertible.

   (c) The set of matrices of the form $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ is a subspace of $M_{2\times 2}$.

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(d) The matrix of the linear transformation T which reflects points in $\mathbb{R}^2$ about the x-axis and then about the y-axis is the same as the matrix of the linear transformation S which rotates points in $\mathbb{R}^2$ about the origin by 180 degrees counterclockwise.

(e) The following set is a basis for P_2: $\{1, 1+t, 1+t+t^2\}$

(f) If V is a set that contains the $\mathbf{0}$-vector, and such that whenever $\mathbf{u}$ and $\mathbf{v}$ are in V, then $\mathbf{u}+\mathbf{v}$ is in V, then V is a vector space!

3. (a) If A and B are square matrices, then $(A+B)^{-1} = A^{-1} + B^{-1}$.

(b) If $T : \mathbb{R}^n \to \mathbb{R}^n$ is a one-to-one linear transformation, then T is also onto.

(c) If $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ are linearly independent vectors in $\mathbb{R}^n$, then $\{\mathbf{v_1}, \mathbf{v_2}\}$ is linearly independent as well!

(d) If A is an invertible square matrix, then $(A^T)^{-1} = (A^{-1})^T$

(e) If A is a 3×3 matrix with two pivot positions, then the equation $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution.

(f) If A and B are square matrices, then $det(A+B) = det(A) + det(B)$.

(g) If $Nul(A) = \{\mathbf{0}\}$, then A is invertible.

(h) $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$

(i) If W is a subspace of V and $\mathcal{B}$ is a basis for V, then some subset of $\mathcal{B}$ is a basis for W.

4. (a) If Nul(A) $= \{\mathbf{0}\}$, then A is invertible

(b) If $AB = I$, then A is invertible

(c) If A is a 2×3 matrix, then $A\mathbf{x} = \mathbf{0}$ always has a nonzero solution

(d) If $T : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is one-to-one, then T is also onto $\mathbb{R}^n$

(e) If A is $n \times n$ and has n pivots, then the columns of A form a basis for $\mathbb{R}^n$

(f) If W is a subspace of V, and $\mathcal{B}$ is a basis for V, then some subset of $\mathcal{B}$ is a basis for W

(g) The intersection of two subspaces of V is a subspace of V

(h) The union of two subspaces of V is a subspace of V

(i) If A and B are symmetric, then so is $AB + B^T A^T$

(j) A system of 10 equations in 8 unknowns always has a nonzero solution