1. (a) If the augmented matrix of the system $Ax = b$ has a pivot in the last column, then the system $Ax = b$ has no solution.

(b) If $A$ and $B$ are invertible $2 \times 2$ matrices, then $(AB)^{-1} = A^{-1}B^{-1}$

(c) If $A$ is a $3 \times 3$ matrix such that the system $Ax = 0$ has only the trivial solution, then the equation $Ax = b$ is consistent for every $b$ in $\mathbb{R}^3$.

(d) The general solution to $Ax = b$ is of the form $x = x_p + x_0$, where $x_p$ is a particular solution to $Ax = b$ and $x_0$ is the general solution to $Ax = 0$.

(e) If $P$ and $D$ are $n \times n$ matrices, then $\det(PDP^{-1}) = \det(D)$

(f) If $T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix}$, then $\text{Nul}(T) = \text{Span}\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$

(g) The set of polynomials $p$ in $P_2$ such that $p(3) = 0$ is a subspace of $P_2$

(h) $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$

2. (a) If $A$ and $B$ are any $2 \times 2$ matrices, then $AB = BA$

(b) The matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 0 & 3 & 0 \end{bmatrix}$ is not invertible.

(c) The set of matrices of the form $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ is a subspace of $M_{2 \times 2}$.

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(d) The matrix of the linear transformation $T$ which reflects points in $\mathbb{R}^2$ about the $x$-axis and then about the $y$-axis is the same as the matrix of the linear transformation $S$ which rotates points in $\mathbb{R}^2$ about the origin by 180 degrees counterclockwise.

(e) The following set is a basis for $P_2$: $\{1, 1 + t, 1 + t + t^2\}$

(f) If $V$ is a set that contains the 0-vector, and such that whenever $u$ and $v$ are in $V$, then $u + v$ is in $V$, then $V$ is a vector space!

3. (a) If $A$ and $B$ are square matrices, then $(A + B)^{-1} = A^{-1} + B^{-1}$.

(b) If $T : \mathbb{R}^n \to \mathbb{R}^n$ is a one-to-one linear transformation, then $T$ is also onto.

(c) If $\{v_1, v_2, v_3\}$ are linearly independent vectors in $\mathbb{R}^n$, then $\{v_1, v_2\}$ is linearly independent as well!

(d) If $A$ is an invertible square matrix, then $(A^T)^{-1} = (A^{-1})^T$.

(e) If $A$ is a $3 \times 3$ matrix with two pivot positions, then the equation $Ax = 0$ has a nontrivial solution.

(f) If $A$ and $B$ are square matrices, then $\det(A + B) = \det(A) + \det(B)$.

(g) If $\text{Nul}(A) = \{0\}$, then $A$ is invertible.

(h) $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$

(i) If $W$ is a subspace of $V$ and $B$ is a basis for $V$, then some subset of $B$ is a basis for $W$.

4. (a) If $\text{Nul}(A) = \{0\}$, then $A$ is invertible

(b) If $AB = I$, then $A$ is invertible

(c) If $A$ is a $2 \times 3$ matrix, then $Ax = 0$ always has a nonzero solu-
(d) If $T : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is one-to-one, then $T$ is also onto $\mathbb{R}^n$

(e) If $A$ is $n \times n$ and has $n$ pivots, then the columns of $A$ form a basis for $\mathbb{R}^n$

(f) If $W$ is a subspace of $V$, and $\mathcal{B}$ is a basis for $V$, then some subset of $\mathcal{B}$ is a basis for $W$

(g) The intersection of two subspaces of $V$ is a subspace of $V$

(h) The union of two subspaces of $V$ is a subspace of $V$

(i) If $A$ and $B$ are symmetric, then so is $AB + B^T A^T$

(j) A system of 10 equations in 8 unknowns always has a nonzero solution