MATH 54 – TRUE/FALSE QUESTIONS FOR MIDTERM 2

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1. (a) If A is diagonalizable, then A^3 is diagonalizable.

   (b) If A is a 3×3 matrix with 3 (linearly independent) eigenvectors, then A is diagonalizable

   (c) If A is a 3×3 matrix with eigenvalues $\lambda = 1, 2, 3$, then A is invertible

   (d) If A is a 3×3 matrix with eigenvalues $\lambda = 1, 2, 3$, then A is (always) diagonalizable

   (e) If A is a 3×3 matrix with eigenvalues $\lambda = 1, 2, 2$, then A is (always) not diagonalizable

   (f) If $\hat{\mathbf{x}}$ is the orthogonal projection of $\mathbf{x}$ on W, then $\hat{\mathbf{x}}$ is orthogonal to $\mathbf{x}$.

   (g) If $\hat{\mathbf{u}}$ is the orthogonal projection of $\mathbf{u}$ on $Span\{\mathbf{v}\}$, then:

$$\hat{\mathbf{u}} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}}\right) \mathbf{u}$$

   (h) If Q is an orthogonal matrix, then Q is invertible.

2. (a) If A is diagonalizable, then A is invertible.

   (b) If A is invertible, then A is diagonalizable

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3. (a) If $\mathcal{A} = \{\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}\}$ and $\mathcal{D} = \{\mathbf{d_1}, \mathbf{d_2}, \mathbf{d_3}\}$ are bases for V, and P is the matrix whose ith column is $[\mathbf{d_i}]_{\mathcal{A}}$, then for all $\mathbf{x}$ in V, we have $[\mathbf{x}]_{\mathcal{D}} = P\,[\mathbf{x}]_{\mathcal{A}}$

   (b) A 3×3 matrix A with only one eigenvalue cannot be diagonalizable

   (c) If $\mathbf{v_1}$ and $\mathbf{v_2}$ are 2 eigenvectors corresponding to 2 **different** eigenvalues λ_1 and λ_2, then $\mathbf{v_1}$ and $\mathbf{v_2}$ are linearly independent!

   (d) If a matrix A has orthogonal columns, then it is an orthogonal matrix.

   (e) For every subspace W and every vector $\mathbf{y}$, $\mathbf{y} - Proj_W\mathbf{y}$ is orthogonal to $Proj_W\mathbf{y}$ (proof by picture is ok here)

   (f) If $\mathbf{y}$ is already in W, then $Proj_W\mathbf{y} = \mathbf{y}$

4. (a) If A is a 3×3 matrix with eigenvalues $\lambda = 0, 2, 3$, then A must be diagonalizable!

   (b) There does not exist a 3×3 matrix A with eigenvalues $\lambda = 1, -1, -1 + i$. (ignore this)

   (c) If A is a symmetric matrix, then all its eigenvectors are orthogonal.

   (d) If Q is an orthogonal $n \times n$ matrix, then $Row(Q) = Col(Q)$.

   (e) The equation $A\mathbf{x} = \mathbf{b}$, where A is a $n \times n$ matrix always has a unique least-squares solution.

   (f) If $AB = I$, then $BA = I$.

   (g) If A is a square matrix, then $Rank(A) = Rank(A^2)$

   (h) If W is a subspace, and $P\mathbf{y}$ is the orthogonal projection of $\mathbf{y}$ onto W, then $P^2\mathbf{y} = P\mathbf{y}$

(i) If $T : V \to W$, where $dim(V) = 3$ and $dim(W) = 2$, then T cannot be one-to-one.

(j) If A is similar to B, then $det(A) = det(B)$.