111 TRUE/FALSE QUESTIONS

Chapter 1: Systems of Linear Equations

(1) A system of 3 linear equations in 2 unknowns must have no solution

(2) A system of 2 linear equations in 3 unknowns could have exactly one solution

(3) A system of linear equations could have exactly two solutions

(4) If there's a pivot in every row of A, then $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$

(5) If the augmented matrix has a pivot in the last column, then $A\mathbf{x} = \mathbf{b}$ is inconsistent

(6) If A has a row of zeros, then $A\mathbf{x} = \mathbf{b}$ is inconsistent for all $\mathbf{b}$

(7) $A\mathbf{x} = \mathbf{0}$ is always consistent

(8) If $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly dependent then $\{A\mathbf{u}, A\mathbf{v}, A\mathbf{w}\}$ is also linearly dependent for every A

(9) If $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent and $\{\mathbf{v}, \mathbf{w}, \mathbf{p}\}$ is linearly independent, then so is $\{\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{p}\}$

(10) If $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly dependent, then $\mathbf{u}$ is in the span of $\{\mathbf{v}, \mathbf{w}\}$

(11) If $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly dependent and $\{\mathbf{u}, \mathbf{v}\}$ is linearly independent, then $\mathbf{w}$ is in the span of $\{\mathbf{u}, \mathbf{v}\}$

(12) If T is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$, then the matrix A of T is a 2×3 matrix

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(13) If $T(c\mathbf{u}) = cT(\mathbf{u})$ for every real number c, then T is a linear transformation

(14) If $T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$ for all $\mathbf{u}$ and $\mathbf{v}$, then T is a linear transformation

(15) If $T(\mathbf{u}+c\mathbf{v}) = T(\mathbf{u})+cT(\mathbf{v})$ for all $\mathbf{u}$ and $\mathbf{v}$ and every real number c, then T is a linear transformation

(16) If $T : \mathbb{R}^2 \to \mathbb{R}^3$ is linear, then T cannot be onto $\mathbb{R}^3$

(17) If T is one-to-one and $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent, then $\{T(\mathbf{u}), T(\mathbf{v}), T(\mathbf{w})\}$ is also linearly independent

Chapter 2: Matrix Algebra

(18) $AB + B^T A^T$ is always symmetric

(19) Any matrix A can be written as a sum of a symmetric ($A^T = A$) and antisymmetric ($A^T = -A$) matrix

(20) $(AB)^{-1} = A^{-1}B^{-1}$

(21) If $AB = AC$, then $B = C$

(22) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ is not invertible

(23) If $AB = I$ for some B, then A is invertible

(24) A 3×2 matrix could be invertible

(25) A 2×3 matrix could be invertible

(26) If AB is invertible, then A and B are invertible

(27) Same, but this time A and B are square

(28) If $Nul(A) = \{\mathbf{0}\}$, then A is invertible

(29) Every linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ has a matrix

(30) If $T : \mathbb{R}^n \to \mathbb{R}^n$ is one-to-one, then T is onto $\mathbb{R}^n$

(31) The row-operations that transform A to I also transform I to A^{-1}

(32) If A is square and $A\mathbf{x} = \mathbf{0}$ implies $\mathbf{x} = \mathbf{0}$, then A is row-equivalent to the identity matrix

Chapter 3: Determinants

(33) In general, $\det(2A) = 2\det(A)$

(34) $\det(A + B) = \det(A) + \det(B)$

(35) If $\det(A^2) + 2\det(A) + \det(I) = 0$, then A is invertible

(36) $\det(A^{-1}) = -\det(A)$

(37) If A^{100} is invertible, then A is invertible

(38) If $\det(A) = 1$ and A has only integer entries, then A^{-1} has integer entries

(39) If $\det(A) = 1$ and A and $\mathbf{b}$ have only integer entries, then the solution $\mathbf{x}$ to $A\mathbf{x} = \mathbf{b}$ has only integer entries

Chapter 4: Vector Spaces and Subspaces

(40) $\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 0\}$ is a subspace of $\mathbb{R}^2$

(41) The union of two subspaces of V is still a subspace of V

(42) The intersection of two subspaces of V is still a subspace of V

(43) Given any basis $\mathcal{B}$ of V, and a subspace W of V, then there is a subset of $\mathcal{B}$ that is a basis of W

(44) $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$

(45) $Nul \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = Span \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$

(46) For a fixed $\mathbf{b} \neq \mathbf{0}$, the set of solutions to $A\mathbf{x} = \mathbf{b}$ is a subspace of $\mathbb{R}^n$

(47) If A is a 4×6 matrix with 2 pivot columns, then $Nul(A) = \mathbb{R}^4$

(48) If A is $m \times n$ and has n pivot columns, then $Nul(A) = \{\mathbf{0}\}$

(49) If A is $m \times n$ and has n pivot columns, then $Col(A) = \mathbb{R}^m$

(50) If A is row-equivalent to B, then $Col(A) = Col(B)$

(51) $Rank(A^2) = Rank(A)$

(52) The set W of polynomials of degree n is a subspace of the set V of polynomials (of any degree)

(53) If A is 5×9, then $Nul(A)$ is at least 4 dimensional

(54) $\left\{\cos^2(t), \sin^2(t), \cos(2t)\right\}$ is linearly dependent

(55) $\mathbb{Z}$ is a subspace of $\mathbb{R}$

(56) If W is a subset of V such that $\mathbf{0}$ (the zero vector in V) is in W and W is closed under addition, then W is a subspace of V

(57) If $\mathbf{0}$ is in W and W is closed under scalar multiplication, then W is a subspace of V

(58) If W is closed under addition and scalar multiplication, then W is a subspace of V

(59) A vector space V is always a subspace of something

(60) If A s row-equivalent to B, then the pivot columns of B form a basis for $Col(A)$

(61) Row-operations preserve the span of the columns of a matrix

(62) Row-operations preserve the linear independence relations of the columns of a matrix

(63) If $\mathcal{B}$ spans a space V, then there is a subset of $\mathcal{B}$ that is a basis for V

(64) If $\mathcal{B} = \{\mathbf{v_1}, \cdots, \mathbf{v_n}\}$ is a linearly independent subset of a $n-$dimensional vector space V, then $\mathcal{B}$ is a basis for V

(65) If $\mathcal{B} = \{\mathbf{v_1}, \cdots, \mathbf{v_n}\}$ is a spanning subset of a $n-$dimensional vector space V, then $\mathcal{B}$ is a basis for V

(66) $\dim(P_4) = 4$

(67) If $\mathcal{B}$ is a basis for $\mathbb{R}^n$ and P is a matrix with the vectors of $\mathcal{B}$ as its columns, then $P\mathbf{x} = [\mathbf{x}]_{\mathcal{B}}$ (the coordinates of $\mathbf{x}$ with respect to $\mathcal{B}$)

(68) If $\mathcal{B} = \{\mathbf{b_1}, \cdots, \mathbf{b_n}\}$ and $\mathcal{C}$ are bases of $\mathbb{R}^n$ and $P = \begin{bmatrix} [\mathbf{b_1}]_{\mathcal{C}} & \cdots & [\mathbf{b_n}]_{\mathcal{C}} \end{bmatrix}$, then $P\,[\mathbf{x}]_{\mathcal{C}} = [\mathbf{x}]_{\mathcal{B}}$

(69) If $\mathcal{B} = \{\mathbf{e_1}, \cdots, \mathbf{e_n}\}$ is the standard basis of $\mathbb{R}^n$, then $[\mathbf{x}]_{\mathcal{B}} = \mathbf{x}$

(70) $Rank(A) = Rank(A^T)$

Chapter 5: Eigenvalues and Eigenvectors

(71) A 3×3 matrix with eigenvalues $\lambda = 1, 2, 4$ must be diagonalizable

(72) A 3×3 matrix with eigenvalues $\lambda = 1, 1, 2$ is never diagonalizable

(73) Every matrix is diagonalizable

(74) If A is similar to B, then $\det(A) = \det(B)$

(75) If A is similar to B, then A and B have the same eigenvalues

(76) If A is diagonalizable, then $\det(A)$ is the product of the eigenvalues of A

(77) If A is similar to B, then A and B have the same eigenvectors

(78) If A is invertible, then A is diagonalizable

(79) If A is diagonalizable, then A is invertible

(80) If A is similar to B, then A^2 is similar to B^2

(81) If A is diagonalizable and invertible, then A^{-1} is diagonalizable

(82) If $\lambda = 0$ is an eigenvalue of A, then A is not invertible

(83) (Nonzero) Eigenvectors corresponding to different eigenvalues of A are linearly independent

(84) Every matrix has a real eigenvalue

(85) Every matrix has a complex eigenvalue

(86) If the characteristic polymomial of A is $\lambda^2 - 3\lambda + 2 = 0$, then $A^2 - 3A + 2I = O$ (the zero-matrix)

Chapter 6: Orthogonality and Least-Squares

(87) If $\hat{\mathbf{x}}$ is the orthogonal projection of $\mathbf{x}$ on a subspace W, then $\hat{\mathbf{x}}$ is perpendicular to $\mathbf{x}$

(88) $\hat{\hat{\mathbf{x}}} = \hat{\mathbf{x}}$

(89) The orthogonal projection of $\mathbf{x}$ on $W^\perp$ is $\mathbf{x} - \hat{\mathbf{x}}$

(90) Every (nonzero) subspace W has an orthonormal basis

(91) $W \cap W^\perp = \{\mathbf{0}\}$

(92) $AA^T\mathbf{x}$ is the projection of $\mathbf{x}$ on $Col(A)$

(93) Same, but the columns of A are orthonormal

(94) $Rank(A^T A) = Rank(A)$

(95) If Q is an orthogonal matrix, then Q is invertible

(96) If Q is a matrix with orthonormal columns, then $\|Q\mathbf{x}\| = \|\mathbf{x}\|$

(97) An orthogonal set without the zero-vector is linearly independent

(98) The orthogonal projection of $\mathbf{v}$ on $W = Span\,\{\mathbf{u}\}$ is $\left(\frac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\right)\mathbf{v}$

(99) An orthogonal matrix has orthogonal columns

(100) If $\hat{\mathbf{x}}$ is a least-squares solution of $A\mathbf{x} = \mathbf{b}$, then $\hat{\mathbf{x}}$ is the orthogonal projection of $\mathbf{x}$ on $Col(A)$.

(101) If $\hat{\mathbf{x}}$ is a least-squares solution of $A\mathbf{x} = \mathbf{b}$, then $A\hat{\mathbf{x}}$ is the point on $Col(A)$ that is closest to $\mathbf{b}$

(102) $A\mathbf{x} = \mathbf{b}$ has only one least-squares solution

(103) If $\|\mathbf{u}+\mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2$, then $\mathbf{u}$ is orthogonal to $\mathbf{v}$ (assume that everything is real)

(104) $\int_0^1 f(x)g(x)dx \leq \left(\int_0^1 (f(x))^2 dx\right)^{\frac{1}{2}} \left(\int_0^1 (g(x))^2 dx\right)^{\frac{1}{2}}$

(105) The product of two orthogonal matrices (it it's defined) is orthogonal

(106) $Col(A)$ is orthogonal to $Nul(A^T)$

Chapter 7: Symmetric Matrices

(107) If A is symmetric, then eigenvectors corresponding to different eigenvalues of A are orthogonal

(108) A symmetric matrix has only real eigenvalues

(109) Linearly independent eigenvectors of a symmetric matrix are orthogonal

(110) If A is symmetric, then it is orthogonally diagonalizable

(111) If A is orthogonally diagonalizable, then it is symmetric