111 TRUE/FALSE QUESTIONS

Chapter 1: Systems of Linear Equations

(1) A system of 3 linear equations in 2 unknowns must have no solution

(2) A system of 2 linear equations in 3 unknowns could have exactly one solution

(3) A system of linear equations could have exactly two solutions

(4) If there’s a pivot in every row of $A$, then $Ax = b$ is consistent for every $b$

(5) If the augmented matrix has a pivot in the last column, then $Ax = b$ is inconsistent

(6) If $A$ has a row of zeros, then $Ax = b$ is inconsistent for all $b$

(7) $Ax = 0$ is always consistent

(8) If $\{u, v, w\}$ is linearly dependent then $\{Au, Av, Aw\}$ is also linearly dependent for every $A$

(9) If $\{u, v, w\}$ is linearly independent and $\{v, w, p\}$ is linearly independent, then so is $\{u, v, w, p\}$

(10) If $\{u, v, w\}$ is linearly dependent, then $u$ is in the span of $\{v, w\}$

(11) If $\{u, v, w\}$ is linearly dependent and $\{u, v\}$ is linearly independent, then $w$ is in the span of $\{u, v\}$

(12) If $T$ is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$, then the matrix $A$ of $T$ is a $2 \times 3$ matrix

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(13) If \( T(cu) = cT(u) \) for every real number \( c \), then \( T \) is a linear transformation

(14) If \( T(u + v) = T(u) + T(v) \) for all \( u \) and \( v \), then \( T \) is a linear transformation

(15) If \( T(u + cv) = T(u) + cT(v) \) for all \( u \) and \( v \) and every real number \( c \), then \( T \) is a linear transformation

(16) If \( T : \mathbb{R}^2 \rightarrow \mathbb{R}^3 \) is linear, then \( T \) cannot be onto \( \mathbb{R}^3 \)

(17) If \( T \) is one-to-one and \( \{u, v, w\} \) is linearly independent, then \( \{T(u), T(v), T(w)\} \) is also linearly independent

Chapter 2: Matrix Algebra

(18) \( AB + B^TA^T \) is always symmetric

(19) Any matrix \( A \) can be written as a sum of a symmetric \((A^T = A)\) and antisymmetric \((A^T = -A)\) matrix

(20) \((AB)^{-1} = A^{-1}B^{-1}\)

(21) If \( AB = AC \), then \( B = C \)

\[
\begin{bmatrix}
1 & 2 & 3 \\
2 & 4 & 6 \\
3 & 6 & 9
\end{bmatrix}
\]

is not invertible

(22) \([1 2 3]

(23) If \( AB = I \) for some \( B \), then \( A \) is invertible

(24) A \( 3 \times 2 \) matrix could be invertible

(25) A \( 2 \times 3 \) matrix could be invertible

(26) If \( AB \) is invertible, then \( A \) and \( B \) are invertible

(27) Same, but this time \( A \) and \( B \) are square

(28) If \( Nul(A) = \{0\} \), then \( A \) is invertible
(29) Every linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ has a matrix

(30) If $T : \mathbb{R}^n \to \mathbb{R}^n$ is one-to-one, then $T$ is onto $\mathbb{R}^n$

(31) The row-operations that transform $A$ to $I$ also transform $I$ to $A^{-1}$

(32) If $A$ is square and $Ax = 0$ implies $x = 0$, then $A$ is row-equivalent to the identity matrix

**Chapter 3: Determinants**

(33) In general, $\det(2A) = 2 \det(A)$

(34) $\det(A + B) = \det(A) + \det(B)$

(35) If $\det(A^2) + 2 \det(A) + \det(I) = 0$, then $A$ is invertible

(36) $\det(A^{-1}) = -\det(A)$

(37) If $A^{100}$ is invertible, then $A$ is invertible

(38) If $\det(A) = 1$ and $A$ has only integer entries, then $A^{-1}$ has integer entries

(39) If $\det(A) = 1$ and $A$ and $b$ have only integer entries, then the solution $x$ to $Ax = b$ has only integer entries

**Chapter 4: Vector Spaces and Subspaces**

(40) $\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 0\}$ is a subspace of $\mathbb{R}^2$

(41) The union of two subspaces of $V$ is still a subspace of $V$

(42) The intersection of two subspaces of $V$ is still a subspace of $V$

(43) Given any basis $B$ of $V$, and a subspace $W$ of $V$, then there is a subset of $B$ that is a basis of $W$

(44) $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$
(45) \( Nul \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = Span \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\} \)

(46) For a fixed \( b \neq 0 \), the set of solutions to \( Ax = b \) is a subspace of \( \mathbb{R}^n \)

(47) If \( A \) is a \( 4 \times 6 \) matrix with 2 pivot columns, then \( Nul(A) = \mathbb{R}^4 \)

(48) If \( A \) is \( m \times n \) and has \( n \) pivot columns, then \( Nul(A) = \{0\} \)

(49) If \( A \) is \( m \times n \) and has \( n \) pivot columns, then \( Col(A) = \mathbb{R}^m \)

(50) If \( A \) is row-equivalent to \( B \), then \( Col(A) = Col(B) \)

(51) \( Rank(A^2) = Rank(A) \)

(52) The set \( W \) of polynomials of degree \( n \) is a subspace of the set \( V \) of polynomials (of any degree)

(53) If \( A \) is \( 5 \times 9 \), then \( Nul(A) \) is at least 4 dimensional

(54) \( \{ cos^2(t), \sin^2(t), \cos(2t) \} \) is linearly dependent

(55) \( \mathbb{Z} \) is a subspace of \( \mathbb{R} \)

(56) If \( W \) is a subset of \( V \) such that 0 (the zero vector in \( V \)) is in \( W \) and \( W \) is closed under addition, then \( W \) is a subspace of \( V \)

(57) If 0 is in \( W \) and \( W \) is closed under scalar multiplication, then \( W \) is a subspace of \( V \)

(58) If \( W \) is closed under addition and scalar multiplication, then \( W \) is a subspace of \( V \)

(59) A vector space \( V \) is always a subspace of something

(60) If \( A \) is row-equivalent to \( B \), then the pivot columns of \( B \) form a basis for \( Col(A) \)

(61) Row-operations preserve the span of the columns of a matrix
(62) Row-operations preserve the linear independence relations of the columns of a matrix

(63) If \( B \) spans a space \( V \), then there is a subset of \( B \) that is a basis for \( V \)

(64) If \( B = \{v_1, \ldots, v_n\} \) is a linearly independent subset of a \( n \)-dimensional vector space \( V \), then \( B \) is a basis for \( V \)

(65) If \( B = \{v_1, \ldots, v_n\} \) is a spanning subset of a \( n \)-dimensional vector space \( V \), then \( B \) is a basis for \( V \)

(66) \( \dim(P_4) = 4 \)

(67) If \( B \) is a basis for \( \mathbb{R}^n \) and \( P \) is a matrix with the vectors of \( B \) as its columns, then \( Px = [x]_B \) (the coordinates of \( x \) with respect to \( B \))

(68) If \( B = \{b_1, \ldots, b_n\} \) and \( C \) are bases of \( \mathbb{R}^n \) and \( P = [[b_1]_C \ldots [b_n]_C] \), then \( P[x]_C = [x]_B \)

(69) If \( B = \{e_1, \ldots, e_n\} \) is the standard basis of \( \mathbb{R}^n \), then \( [x]_B = x \)

(70) \( \text{Rank}(A) = \text{Rank}(A^T) \)

Chapter 5: Eigenvalues and Eigenvectors

(71) A \( 3 \times 3 \) matrix with eigenvalues \( \lambda = 1, 2, 4 \) must be diagonalizable

(72) A \( 3 \times 3 \) matrix with eigenvalues \( \lambda = 1, 1, 2 \) is never diagonalizable

(73) Every matrix is diagonalizable

(74) If \( A \) is similar to \( B \), then \( \det(A) = \det(B) \)

(75) If \( A \) is similar to \( B \), then \( A \) and \( B \) have the same eigenvalues

(76) If \( A \) is diagonalizable, then \( \det(A) \) is the product of the eigenvalues of \( A \)

(77) If \( A \) is similar to \( B \), then \( A \) and \( B \) have the same eigenvectors
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(78) If $A$ is invertible, then $A$ is diagonalizable

(79) If $A$ is diagonalizable, then $A$ is invertible

(80) If $A$ is similar to $B$, then $A^2$ is similar to $B^2$

(81) If $A$ is diagonalizable and invertible, then $A^{-1}$ is diagonalizable

(82) If $\lambda = 0$ is an eigenvalue of $A$, then $A$ is not invertible

(83) (Nonzero) Eigenvectors corresponding to different eigenvalues of $A$ are linearly independent

(84) Every matrix has a real eigenvalue

(85) Every matrix has a complex eigenvalue

(86) If the characteristic polynomial of $A$ is $\lambda^2 - 3\lambda + 2 = 0$, then $A^2 - 3A + 2I = O$ (the zero-matrix)

Chapter 6: Orthogonality and Least-Squares

(87) If $\hat{x}$ is the orthogonal projection of $x$ on a subspace $W$, then $\hat{x}$ is perpendicular to $x$

(88) $\hat{x} = \hat{x}$

(89) The orthogonal projection of $x$ on $W^\perp$ is $x - \hat{x}$

(90) Every (nonzero) subspace $W$ has an orthonormal basis

(91) $W \cap W^\perp = \{0\}$

(92) $AA^T x$ is the projection of $x$ on $Col(A)$

(93) Same, but the columns of $A$ are orthonormal

(94) $\text{Rank}(A^T A) = \text{Rank}(A)$

(95) If $Q$ is an orthogonal matrix, then $Q$ is invertible
(96) If $Q$ is a matrix with orthonormal columns, then $\|Qx\| = \|x\|$

(97) An orthogonal set without the zero-vector is linearly independent

(98) The orthogonal projection of $v$ on $W = \text{Span} \{u\}$ is $\left( \frac{u \cdot v}{v \cdot v} \right) v$

(99) An orthogonal matrix has orthogonal columns

(100) If $\hat{x}$ is a least-squares solution of $Ax = b$, then $\hat{x}$ is the orthogonal projection of $x$ on $Col(A)$.

(101) If $\hat{x}$ is a least-squares solution of $Ax = b$, then $A\hat{x}$ is the point on $Col(A)$ that is closest to $b$

(102) $Ax = b$ has only one least-squares solution

(103) If $\|u + v\|^2 = \|u\|^2 + \|v\|^2$, then $u$ is orthogonal to $v$ (assume that everything is real)

(104) $\int_0^1 f(x)g(x)dx \leq \left( \int_0^1 (f(x))^2dx \right)^{\frac{1}{2}} \left( \int_0^1 (g(x))^2dx \right)^{\frac{1}{2}}$

(105) The product of two orthogonal matrices (it it’s defined) is orthogonal

(106) $Col(A)$ is orthogonal to $Nul(A^T)$

**Chapter 7: Symmetric Matrices**

(107) If $A$ is symmetric, then eigenvectors corresponding to different eigenvalues of $A$ are orthogonal

(108) A symmetric matrix has only real eigenvalues

(109) Linearly independent eigenvectors of a symmetric matrix are orthogonal

(110) If $A$ is symmetric, then it is orthogonally diagonalizable

(111) If $A$ is orthogonally diagonalizable, then it is symmetric