MONDAY, APRIL 1, 2019

LECTURE 1 - VECTOR SPACES

I - INTRODUCTION

) ALRIGHT, LET'S GET THIS MATH PARTY STARTED!
HELLO EVERYONE, AND WELCOME TO MATH 121A!
MY NAME IS PEYAM AND I'LL BE YOUR INSTRUCTOR THIS QUARTER
NOTICE THE π IN MY NAME, THAT'S B/C I ♡ MATH (π)
AND I ♡ FOOD (π)

BTW, I KNOW PEOPLE LIKE TO CALL ME PROF. TABRIZIAN,
BUT PLEASE CALL ME PEYAM, OR DR PEYAM, LIKE MY AWESOME
YOUTUBE CHANNEL, WHICH HIT 19K SUBSCRIBERS LAST WEEK

) LOGISTICS: ALL THE INFO IS ON THE SYLLABUS

COURSE: THIS COURSE MEETS MWF HERE, BUT WHETHER OR NOT YOU SHOW UP IS UP TO YOU
ALSO DISCUSSION SECTION ON MW, TO GET YOUR DAILY DOUBLE DOSE OF LA

OH: TU 2-3, W 1-2 IN 410N (ANYTIMES WITH 410)
OK TO SHOW UP @ OTHER TIMES, BUT NOT FOR TOO LONG

TEXTBOOK: OMG, MY FAAAVORITE TEXTBOOK ♡
SAME ONE I USED BACK IN 2007 WHEN I TOOK LA! YOU'LL HATE IT, BUT YOU'LL LEARN TO LOVE IT, TRUST ME

⚠ I'M EXPECTING YOU TO READ THE TEXTBOOK CAREFULLY
THIS IS UPPER-DIVISION MATH, DON'T EXPECT ME TO COVER EVERYTHING IN LECTURE

GRADING

HW 20% — LOTS OF HW, BUT ALSO LEARN A LOT
DUE ON W DURING DISCUSSION
HW 1 DUE THIS W
10 HW, LOWEST 2 DROPPED

QUIZZES 10% — EVERY W DURING DISCUSSION, BASED ON HW
QUIZ 1 THIS W
10 QUIZZES, LOWEST 2 DROPPED

MIDTERM 25% — F, MAY 3 (DURING LECTURE)

FINAL 45% — TU, JUNE 11, 1:30 – 3:30 PM

GRADES — THIS CLASS IS VERY HARD, BUT NO MATTER WHAT HAPPENS, THIS CLASS IS GOING TO BE CURVED

MATH DEPT CURVE: 20% A, 25% B, 30% C, 15% D, 10% F

II – MOTIVATION

TODAY: WE'LL LAY THE GROUNDWORK OF OUR LA ADVENTURE BY DEFINING THE NOTION OF A VECTOR SPACE
(IF LA = WAR, THEN VS = BATTLEFIELD)
LET ME FIRST GIVE SOME MOTIVATION

EX: VECTORS IN $\mathbb{R}^N$, $x = \begin{bmatrix} a_1 \\ \vdots \\ a_N \end{bmatrix}$

ADDITION: $\begin{bmatrix} a_1 \\ \vdots \\ a_N \end{bmatrix} + \begin{bmatrix} b_1 \\ \vdots \\ b_N \end{bmatrix} = \begin{bmatrix} a_1 + b_1 \\ \vdots \\ a_N + b_N \end{bmatrix}$ "$x + y$"

SCALAR MULTIPLICATION: $c \begin{bmatrix} a_1 \\ \vdots \\ a_N \end{bmatrix} = \begin{bmatrix} c a_1 \\ \vdots \\ c a_N \end{bmatrix}$ "cx"

BIG QUESTION: WHAT IF WE FORGET ALL ABOUT VECTORS, EXCEPT THAT WE CAN ADD AND SCALAR MULTIPLY THEM?

(AND THIS LEADS TO THE NOTION OF A VS)

III – DEFINITION

NOTE: IN THIS COURSE, $\mathbb{F} = \mathbb{R}$ OR $\mathbb{C}$

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DEF A VECTOR SPACE V OVER $\mathbb{F}$ IS A SET WITH TWO OPERATIONS: ADDITION $(+)$ AND SCALAR MULTIPLICATION $(\cdot)$ SUCH THAT THE FOLLOWING 10 PROPERTIES ARE SATISFIED:

1) IF x & y ARE IN V, $x+y$ IS IN V ("CLOSED UNDER +")

2) IF x ~~IS~~ IN V AND c IS IN $\mathbb{F}$, cx IS IN V ("CLOSED UNDER ·")

"FOR ALL"

3) $\forall x, y$ IN V, $x+y = y+x$ (COMMUTATIVITY)

4) $\forall x, y, z$ IN V, $(x+y)+z = x+(y+z)$ (ASSOCIATIVITY)

5) THERE IS A ZERO VECTOR $\underline{0}$ IN V SUCH THAT $x+\underline{0} = \underline{0}+x = x$ $\forall x$ IN V

6) $\forall x$ IN V THERE IS AN ELEMENT $-x$ IN V SUCH THAT $x+(-x) = \underline{0}$

7) $\forall x$ IN V, $1x = x$

8) $\forall a, b$ IN $\mathbb{F}$ AND x IN V, $a(bx) = (ab)x$

$2(3x) = 6x$

9) $\forall a$ IN $\mathbb{F}$ AND x, y IN V, $a(x+y) = ax+ay$

10) $\forall a, b$ IN $\mathbb{F}$ AND x IN V, $(a+b)x = ax+bx$

("DISTRIBUTIVITY")

IT'S QUITE A MOUTHFUL, BUT HOPEFULLY THESE PROPERTIES ARE "OBVIOUS"

THESE ARE THE PROPERTIES THAT MAKE $\mathbb{R}^n$ WORK

IV - EXAMPLES

THERE ARE MANY SETS OTHER THAN $\mathbb{R}^N$ THAT ARE VS (SEE BOOK OR YOUTUBE FOR MORE EXAMPLES)

EX 1 $\mathbb{R}^N$ (SEE MOTIVATION)

EX 2 $M_{M \times N}$ (M×N MATRICES)

EX $M_{2\times3}$ $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 2 \\ 4 & 6 & 7 \end{bmatrix}$

(SO MATRICES ARE VECTORS IN OUR COURSE)

EX 3 P_N = POLYNOMIALS OF DEGREE $\leq N$

$$p(x) = a_N x^N + a_{N-1} x^{N-1} + \cdots + a_1 x + a_0$$

EX P_3 $2x^3 + 3x^2 - x + 1$

EX 4 $\mathcal{F}(\mathbb{R}, \mathbb{R})$ = FUNCTIONS FROM $\mathbb{R}$ TO $\mathbb{R}$

EX e^x, $\sin(x)$, ...

$(f+g)(x) = f(x) + g(x)$
$(cf)(x) = c(f(x))$ (AND MANY MORE, 24 EX ON YT)

V - SOME CONSEQUENCES

USING THESE PROPERTIES, WE CAN SHOW SOME NEAT CONSEQUENCES ARE TRUE

⚠ TODAY WE'RE EXTREMELY PICKY
STARTING ON W, WE'LL ASSUME ALL THOSE PROPERTIES ARE TRUE

EX SHOW THAT $\forall x, y, z$ IN V,

$$x + z = y + z \Rightarrow x = y$$ "CANCELLATION LAW"

$$x + z = y + z$$

$$\Rightarrow (x+z)+(-z) = (y+z)+(-z)$$ ASSOCIATIVITY

$$\Rightarrow x + (z+(-z)) = y + (z+(-z))$$ DEF OF $-z$

$$\Rightarrow x + \underline{0} = y + \underline{0}$$ DEF OF $\underline{0}$

$$\Rightarrow x = y \quad \blacksquare$$

EX SHOW THAT $\forall x$ IN V, $0x = \underline{0}$

TRICK CONSIDER $(0+0)x$

SINCE $0+0=0$ (in F), $(0+0)x = 0x$

ON THE OTHER HAND, BY DISTRIBUTIVITY,

$$(0+0)x = 0x + 0x$$

So $0x + 0x = (0+0)x = 0x$

$0x + \cancel{0x} = \underline{0} + \cancel{0x}$ (DEF OF $\underline{0}$)

So $0x = \underline{0}$ (CANCELLATION LAW) ■

EX SHOW $(-a)x = -(ax)$

ENOUGH TO SHOW $(-a)x + ax = \underline{0}$ ($\Rightarrow (-a)x = -ax$)

But $(-a)x + ax = ((-a)+a)x$ (DISTRIBUTIVITY)

$= 0x$ ($-a+a=0$ in $\mathbb{F}$)

$= \underline{0}$ (ABOVE) ■

EX SHOW $\underline{0}$ IS UNIQUE

SUPPOSE THERE ARE TWO ZERO VECTORS $\underline{0}$ AND $\underline{0}'$

BY DEF OF $\underline{0}$, $\underline{0} + \underline{0}' = \underline{0}'$ ($\underline{0} + x = x$)

BY DEF OF $\underline{0}'$, $\underline{0} + \underline{0}' = \underline{0}$ ($x + \underline{0}' = x$)

So $\underline{0} = \underline{0} + \underline{0}' = \underline{0}' \Rightarrow \underline{0} = \underline{0}'$ ■