AP SOLUTIONS (HW #1)

i) FIRST OF ALL, IF X IS IN $V = \mathbb{R}$ AND Y IS IN V, THEN

$$X \oplus Y = X + Y - 1, \text{ WHICH IS IN } V$$

ii) AND IF X IS IN V AND C IS IN $\mathbb{R}$, THEN

$$C \odot X = CX + (1 - C), \text{ WHICH IS IN } V$$

(VS 1) SUPPOSE X AND Y ARE IN V, THEN

$$X \oplus Y = X + Y - 1 = Y + X - 1 = Y \oplus X \checkmark$$

(VS 2) SUPPOSE X, Y, Z ARE IN V, THEN

$$(X \oplus Y) \oplus Z = (X + Y - 1) \oplus Z$$
$$= (X + Y - 1) + Z - 1$$
$$= X + Y + Z - 2$$

AND $X \oplus (Y \oplus Z) = X + (Y \oplus Z) - 1$

$$= X + (Y + Z - 1) - 1$$
$$= X + Y + Z - 2$$

SO $(X \oplus Y) \oplus Z = X \oplus (Y \oplus Z)$

(VS 3) [PREP LET'S TRY TO FIGURE OUT WHAT THE ZERO VECTOR IS!

LET X BE IN V, THEN:

$x \oplus \underline{0} = x \implies \cancel{x} + \underline{0} - 1 = \cancel{x} \implies \underline{0} = 1$

SO THE $\underline{0}$ VECTOR IN V IS JUST $\underline{1}$, HOW COOL IS THAT!]

LET x BE IN V, THEN

$x \oplus \underline{0} = x \oplus 1 = (x + \cancel{1}) - \cancel{1} = x$ ✓

(VS 4) [PREP LET'S FIND $-x$:

LET x BE IN V, THEN $x \oplus y = \underline{0}$

$x \oplus y = \underline{0} \implies x + y - 1 = 1 \implies x + y = 2 \implies y = 2 - x$

SO $-x$ IS $2-x$]

LET x BE IN V, AND $y = 2-x$ IN V, THEN

$x \oplus y = x + y - 1 = \cancel{x} + 2 - \cancel{x} - 1 = 1 = \underline{0}$, SO $x \oplus y = \underline{0}$ ✓

(VS 5) LET x BE IN V, THEN

$1 \odot x = 1x + (1-1) = x$ ✓

(VS 6) LET a, b IN $\mathbb{R}$ AND x IN V, THEN

$(ab) \odot x = abx + (1 - ab)$

AND $a \odot (b \odot x) = a(b \odot x) + 1 - a$

$= a(bx + 1 - b) + 1 - a$

$= abx + a(1-b) + 1 - a$

$= abx + \cancel{a} - ab + 1 - \cancel{a} = abx + (1 - ab) = (ab) \odot x$

(VS 7) LET a BE IN $\mathbb{R}$ AND x, y IN V, THEN

$$a \odot (x \oplus y) = a(x \oplus y) + (1-a)$$
$$= a(x+y-1) + 1-a$$
$$= ax + ay - a + 1 - a$$
$$= ax + ay + 1 - 2a$$

$$(a \odot x) \oplus (a \odot y) = (a \odot x) + (a \odot y) - 1$$
$$= ax + (1-a) + ay + (\cancel{1}-a) - \cancel{1}$$
$$= ax + ay + 1 - 2a = a \odot (x \oplus y) \checkmark$$

(VS 8) LET a, b IN $\mathbb{R}$ AND x IN V, THEN

$$(a+b) \odot x = (a+b)x + (1-(a+b))$$
$$= ax + bx + 1 - a - b$$

$$(a \odot x) \oplus (b \odot x) = (a \odot x) + (b \odot x) - 1$$
$$= ax + (1-a) + bx + (\cancel{1}-b) - \cancel{1}$$
$$= ax + bx + 1 - a - b$$
$$= (a+b) \odot x$$

HENCE V IS A VECTOR SPACE OVER $\mathbb{R}$.