1· 2.1 (a) True. Property 3 of vector spaces.

(b) False. Assume that there are two zero vectors, $\vec{0}$ and $\vec{0}'$. Then given any vector $\mathbf{x}$, $\mathbf{x}+\vec{0}=\mathbf{x}=\mathbf{x}+\vec{0}'$ by property of the zero vector. Then by the Cancellation Law for Vector Addition (page 11) $\vec{0}=\vec{0}'$.

(c) False. $\mathbf{x}$ could be the zero vector.

(d) False. a could be 0.

(e) True.

(f) False. An $m \times n$ matrix has m rows and n columns.

(g) False. Any two polynomials may be added.

(h) False. $f+g$ is a polynomial of degree less than or equal to n.

(i) True.

(j) True.

(k) True.

1· 2.4 (a)

$$\begin{pmatrix} 2 & 5 & -3 \\ 1 & 0 & 7 \end{pmatrix} + \begin{pmatrix} 4 & -2 & 5 \\ -5 & 3 & 2 \end{pmatrix} = \begin{pmatrix} 6 & 3 & 2 \\ -4 & 3 & 9 \end{pmatrix}$$

(c)

$$4\begin{pmatrix} 2 & 5 & -3 \\ 1 & 0 & 7 \end{pmatrix} = \begin{pmatrix} 8 & 20 & -12 \\ 4 & 0 & 28 \end{pmatrix}$$

(e) $(2x^4-7x^3+4x+3)+(8x^3+2x^2-6x+7)=2x^4+x^3+2x^2-2x+10$

1· 2.7

$$\begin{aligned} f(0) &= 2(0)+1=1 & f(1) &= 2(1)+1=3 \\ g(0) &= 1+4(0)-2(0)^2=1 & g(1) &= 1+4(1)-2(1)^2=3 \\ h(0) &= 5^0+1=2 & h(1) &= 5^1+1=6 \end{aligned}$$

Therefore, $f=g$ and $f+g=h$ on the set $S=\{0,1\}$.

1.2.9 (a) **Corollary 1.** The vector $\vec{0}$ described in VS3 is unique.
Assume that there are two zero vectors, $\vec{0}$ and $\vec{0}'$. Then given any vector $\mathbf{x}$, $\mathbf{x}+\vec{0}=\mathbf{x}=\mathbf{x}+\vec{0}'$ by property of the zero vector. Then by the Cancellation Law for Vector Addition (page 11) $\vec{0}=\vec{0}'$.

(b) **Corollary 2.** The vector $\vec{y}$ described in VS4 is unique.
Given a vector $\vec{x}\in V$, let $\vec{y}$ and $\vec{y'}$ satisfy the property in VS4. Then

$$\vec{x}+\vec{y}=\vec{0}=\vec{x}+\vec{y'}.$$

By the Cancellation Law for Vector Addition, we can cancel $\vec{x}$, and so $\vec{y}=\vec{y'}$.

(c) **Theorem 1.2(c)** $a\cdot\vec{0}=\vec{0}$ for all $a\in F$.

$$a\cdot\vec{0}=a\cdot(\vec{0}+\vec{0})=a\cdot\vec{0}+a\cdot\vec{0}$$

and by the Cancellation Law, $0=a\cdot\vec{0}$.

1.2.17 V is not a vector space over F with these operations because $(VS5)$ fails.

$$1\cdot(a_1,a_2)=(a_1,0)\neq(a_1,a_2).$$

1.2.18 V is not a vector space over F with these operations because $(VS1)$ fails.

$$(a_1,a_2)+(b_1,b_2)=(a_1+2b_1,a_2+3b_2)\neq(b_1+2a_1,b_2+3a_2)=(b_1,b_2)+(a_1,a_2).$$

1.2.21 Since addition and multiplication is done component-wise, and each component satisfies $(VS1)-(VS8)$, Z satisfies all the properties and is a vector space.