

AP SOLUTIONS

(HW #4)

$$\begin{aligned} \text{AP1) } \quad T(cx, cy) &= \sqrt[3]{(cx)^3 + (cy)^3} \\ &= \sqrt[3]{c^3(x^3 + y^3)} \\ &= \sqrt[3]{c^3} \sqrt[3]{x^3 + y^3} = c \sqrt[3]{x^3 + y^3} \\ &= c T(x, y) \end{aligned}$$

but T IS NOT LINEAR BECAUSE, FOR EXAMPLE,

$$T(1, 0) + T(0, 1) = 1 + 1 = 2$$

$$\text{but } T(1, 1) = \sqrt[3]{2} \neq 2$$

AP2) (a) T ONE-TO-ONE

$$\text{IFF } T(a_1, \dots, a_N) = \underline{0} \Rightarrow (a_1, \dots, a_N) = (0, \dots, 0)$$

$$\text{IFF } a_1 v_1 + \dots + a_N v_N = \underline{0} \Rightarrow a_1 = 0, \dots, a_N = 0$$

$$\text{IFF } \{v_1, \dots, v_N\} \text{ LI}$$

(b) T IS ONTO V

$$\text{IFF } \forall v \in V \quad v = T(a_1, \dots, a_N) \text{ FOR SOME } (a_1, \dots, a_N) \in \mathbb{F}^N$$

IFF $\forall v \in V, \quad v = a_1 v_1 + \dots + a_n v_n$ For some $a_1, \dots, a_n \in \mathbb{F}$

IFF $\text{span}(\{v_1, \dots, v_n\}) = V$