Math 121A — Homework 1

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**Reading:** Section 1.2. You can read section 1.1 if you want, it’s a gentle introduction to vectors in $\mathbb{R}^n$. In case you’re curious about fields, you might want to check out Appendix C, although in our class, fields will just be $\mathbb{R}$ or $\mathbb{C}$.

**Note:** Make sure to write in complete sentences; you are graded not only on your answer, but also on your work. The problems marked optional are for extra practice; do not turn those in with your homework set. AP refers to the additional problem below. Finally, for the True/False on the homework, you do not need to justify your answer, but on the exam you might have T/F with justification.

- **Section 1.2:** 1, 7, 9, 17, 18, 20 (Optional: 8, 19, AP)

**Optional Additional Problem:** Let $V = \mathbb{R}$, but this time with new operations Peyamaddition $\oplus$ and Peyamultiplication $\odot$ defined by:

\[ x \oplus y = x + y - 1 \]

\[ c \odot x = cx + (1 - c) \]

Show that $V$ is a vector space over $\mathbb{R}$ under those two operations. What is cool is in that vector space, the zero vector $0$ is $1$.

**Warning:** Here the zero-vector $0$ is not the real number $0$. Similarly the additive inverse of $x$ is not the real number $-x$. Figure out what they are. Also, be careful not to confuse scalar multiplication $\odot$ with multiplication in $\mathbb{R}$. For example, for (VS 6), you have to prove that $(ab) \odot x = a \odot (b \odot x)$, where $ab$ is regular.
multiplication of $a$ and $b$. This problem is probably one of the most abstract problems in the course, this is why it’s optional. It’ll get better after that ;)}