Math 121A – Homework 3

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Reading: Sections 1.6 and 2.1. Beware that, although I didn't cover everything from 1.6 during lecture, unless otherwise stated, you're still responsible for knowing all the theorems in this section, including their proofs. Definitely know the statement of the Replacement Theorem (Theorem 1.10), but you can ignore its proof. Also, ignore the section on the Lagrange Interpolation Formula. I'll continue on 2.1 on Wednesday, and there will be more 2.1 problems on Homework 4.

Note: In order to reduce your workload a bit, I'll try my best to cap homework assignments to at most 15 problems a week (including T/F problems). Some of the problems I originally wanted to assign are in the optional section, so try to look at them if you have time.

- **Section 1.6:** 1, 3(c), 7, 14, 16, 19, 20, 29, AP1 (Optional: 4, 12, 15, 21, 26)
- **Section 2.1:** 1, 2, 5, 9(a)(d), 11, 13 (Optional: 9(b), 12, AP2)

Extra hints for 1.6.29(a): See the definition on page 22. Show that

$$\{u_1, \cdots, u_m, v_1, \cdots, v_m, w_1, \cdots, w_p\}$$

is a basis for $W_1 + W_2$. For the linear independence part, you'll have to notice that $u_1, \cdots, u_k, v_1, \cdots, v_m$ are in W_1, but $w_1, \cdots, w_p$ are not in W_1, and derive some sort of contradiction.

Additional Problem 1: Let S_1 and S_2 be finite subsets of a vector space V, with S_1 linearly independent, $Span(S_2) = V$, and S_2 has the same number of elements as S_1. Use the Replacement Theorem to show that S_1 spans V.

Additional Problem 2: Let V and W be vector spaces and let $T : V \to W$ be a function from V to W (not necessarily a linear transformation). Define the graph G of T to be

$$G = \{(v, T(v)) \in V \times W \mid v \in V\}$$

Show that T is a linear transformation if and only if its graph G is a subspace of $V \times W$.

Note: This is why sometimes vector spaces are called *linear* spaces.