Math 121A – Homework 4

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Reading: Sections 2.1, 2.2, 2.3. In 2.3, ignore the section on Applications. There will be more 2.3 problems on Homework 5.

- **Section 2.1:** 14(c), 15, 17, 21, 28 (Optional: 16, 20, 37, 39, AP1, AP2)
- **Section 2.2:** 1, 5(b)(d)(f), 8, 13, 14, 16 (Optional: 5(a)(c)(e)(g), 10)
- **Section 2.3:** 1, 3, 11

Hint for 2.2.14: Apply your equality to the vectors $x, x^2, \cdots, x^n$ respectively

Hint for 2.2.16: This is similar in spirit to the proof of the rank-nullity theorem. Let $\{\mathbf{v_1}, \cdots, \mathbf{v_m}\}$ be a basis for $N(T)$, and extend this to a basis $\beta = \{\mathbf{v_1}, \cdots, \mathbf{v_m}, \mathbf{v_{m+1}}, \cdots, \mathbf{v_n}\}$ for V. Consider $\{T(\mathbf{v_{m+1}}), \cdots, T(\mathbf{v_n})\}$ (shown in class to be a basis for $R(T)$), and extend this to a basis $\gamma = \{\mathbf{w_1}, \cdots, \mathbf{w_m}, T(\mathbf{v_{m+1}}), \cdots, T(\mathbf{v_n})\}$ of W. Now calculate $[T]_\beta^\gamma$.

Additional Problem 1: Define $T : \mathbb{R}^2 \to \mathbb{R}$ by $T(x, y) = \sqrt[3]{x^3 + y^3}$. Show that $T(c\mathbf{u}) = cT(\mathbf{u})$ but T is not linear

Additional Problem 2: Suppose $\mathbf{v_1}, \cdots, \mathbf{v_n}$ are fixed vectors in V, and define $T : \mathbb{F}^n \to V$:

$$T(a_1, \cdots, a_n) = a_1\mathbf{v_1} + \cdots + a_n\mathbf{v_n}$$

(a) Show T is one-to-one if and only if $\{\mathbf{v_1}, \cdots, \mathbf{v_n}\}$ is linearly independent

(b) Show T is onto V if and only if $\{\mathbf{v_1}, \cdots, \mathbf{v_n}\}$ spans V