MOCK MIDTERM

Instructions: This is a mock midterm, designed to give you some practice for the actual midterm. It will be similar in length and in difficulty to the actual midterm, but beware that the actual exam might have different questions. So please also look at the study guide and the homework for a more complete study experience.

WARNING: On the exam, you are graded not only on the correct answer, but also on the way you write out your answer. Don't be surprised if you get lots of points off if your proof is too short or if you skip too many details! I will be especially picky about this, even more so than the homework grader or your TA or other instructors you may have had. Please look at the solutions to this exam to get an idea of how I want you to write out your answers.

1		10
2		30
3		30
4		30
Total		100

Date: Friday, May 3, 2019.

1. *(10 points)* Find a subset of S that is a basis for $Span(S)$, where

$$S = \{(1,0,0),(2,0,0),(1,1,1),(2,1,1),(1,1,0)\}$$

2. *(30 = 10 + 5 + 15 points)* Let V be a vector space and S be a subset of V

   (a) Define what it means for S to be linearly dependent
   (b) Give an example of a linearly dependent set $S = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$, of nonzero vectors, but where $\mathbf{v_3}$ is not a linear combination of $\mathbf{v_1}$ and $\mathbf{v_2}$
   (c) Show that S is linearly dependent if and only if there are distinct vectors $\mathbf{u}, \mathbf{u_1}, \cdots, \mathbf{u_n}$ in S such that $\mathbf{u}$ is a linear combination of $\mathbf{u_1}, \cdots, \mathbf{u_n}$

3. *(30 = 10 + 20 points)* Let V and W be finite-dimensional vector spaces

   (a) Define what it means for V and W to be isomorphic.
   (b) Show that V and W are isomorphic if and only if $\dim(V) = \dim(W)$.

4. *(30 points)* Let V and W be finite-dimensional vector spaces, let Z be a subspace of V, and suppose $U : Z \to W$ is linear.

   Show that there exists a linear transformation $T : V \to W$ (called an extension of U) such that $T(z) = U(z)$ for all z in Z.