AP 4 SOLUTION

(HW #6)

AP4 (a) SUPPOSE β AND γ ARE TWO BASES OF V

THEN WE KNOW THAT

$$[T]_\gamma^\gamma = Q\,[T]_\beta^\beta\,Q^{-1} \quad \text{FOR SOME INVERTIBLE } Q$$

NAMELY $Q = {}_\gamma Q_\beta$

THEREFORE $[T]_\gamma^\gamma$ AND $[T]_\beta^\beta$ ARE SIMILAR

SO BY PROBLEM 10, $Tr([T]_\gamma^\gamma) = Tr([T]_\beta^\beta)$

(b) LET $\beta = \{1, x, x^2\}$ BE THE STANDARD BASIS OF P_2

$T(1) = 0 = \underline{\underline{0}}\,1 + \underline{\underline{0}}\,x + \underline{\underline{0}}\,x^2$

$T(x) = 1 = \underline{\underline{1}}\,1 + \underline{\underline{0}}\,x + \underline{\underline{0}}\,x^2$

$T(x^2) = 2x = \underline{\underline{0}}\,1 + \underline{\underline{2}}\,x + \underline{\underline{0}}\,x^2$

SO $[T]_\beta^\beta = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

HENCE $Tr(T) = Tr([T]_\beta^\beta) = Tr\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} = 0 + 0 + 0 = \boxed{0}$