Math 110, Professor Ogus, Homework due 3/7
(written by Janak Ramakrishnan)

**4.1.1.** (a) F, (b) T, (c) F, (d) F, (e) F, (f) F, (g) T

**4.1.3b.** $(5-2i)(7i)-(6+4i)(-3+i) = 35i+14+18+12i-6i+4 = 36+41i$.

**4.1.7.** Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Then $\det(A^t) = ad - cb = ad - bc = \det(A)$.

**4.2.1.** (a) F, (b) T, (c) T, (d) T, (e) F, (f) F, (g) F, (h) T.

**4.2.4.**

$$\det\begin{pmatrix} b_1+c_1 & b_2+c_2 & b_3+c_3 \\ a_1+c_1 & a_2+c_2 & a_3+c_3 \\ a_1+b_1 & a_2+b_2 & a_3+b_3 \end{pmatrix} = \det\begin{pmatrix} b_1 & b_2 & b_3 \\ a_1+c_1 & a_2+c_2 & a_3+c_3 \\ a_1+b_1 & a_2+b_2 & a_3+b_3 \end{pmatrix} + \det\begin{pmatrix} c_1 & c_2 & c_3 \\ a_1+c_1 & a_2+c_2 & a_3+c_3 \\ a_1+b_1 & a_2+b_2 & a_3+b_3 \end{pmatrix} =$$

$$\det\begin{pmatrix} b_1 & b_2 & b_3 \\ a_1+c_1 & a_2+c_2 & a_3+c_3 \\ b_1 & b_2 & b_3 \end{pmatrix} + \det\begin{pmatrix} b_1 & b_2 & b_3 \\ a_1+c_1 & a_2+c_2 & a_3+c_3 \\ a_1 & a_2 & a_3 \end{pmatrix} +$$

$$\det\begin{pmatrix} c_1 & c_2 & c_3 \\ c_1 & c_2 & c_3 \\ a_1+b_1 & a_2+b_2 & a_3+b_3 \end{pmatrix} + \det\begin{pmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ a_1+b_1 & a_2+b_2 & a_3+b_3 \end{pmatrix} =$$

$$0 + \det\begin{pmatrix} b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \\ a_1 & a_2 & a_3 \end{pmatrix} + \det\begin{pmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \end{pmatrix} + 0 + \det\begin{pmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ a_1 & a_2 & a_3 \end{pmatrix} + \det\begin{pmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix} =$$

$$0 + (-1)^2 \det\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} + 0 + (-1)^2 \det\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} = 2\det\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}.$$

Thus, $k = 2$.

**4.2.10.**

$$\det\begin{pmatrix} \cancel{i} & \cancel{2+i} & \cancel{0} \\ -1 & 3 & 2i \\ 0 & -1 & 1-i \end{pmatrix} = (-1)(-1)(2+i)(1-i) + 3i(1-i) - 2i(-i) = 4+2i$$

**4.2.25.** $kA = kIA = (kI)A$. Thus, $\det(kA) = \det((kI)A) = \det(kI)\det(A)$. Since $kI$ is upper-triangular, $\det(kI)$ is the product of the diagonal entries, which is $k^n$, so $\det(kA) = k^n \det(A)$.

**4.3.1.** (a) F, (b) T, (c) F, (d) T, (e) F, (f) T, (g) F, (h) F.

**4.3.5.**

$$x_1 = \det\begin{pmatrix} -4 & -1 & 4 \\ 8 & 3 & 1 \\ 0 & -1 & 1 \end{pmatrix} = -20, \qquad x_2 = \det\begin{pmatrix} 1 & -4 & 4 \\ -8 & 8 & 1 \\ 2 & 0 & 1 \end{pmatrix} = -48, \qquad x_3 = \det\begin{pmatrix} 1 & -1 & -4 \\ -8 & 3 & 8 \\ 2 & -1 & 0 \end{pmatrix} = -8$$

**4.3.21.** By performing elementary row operations on $C$, we can reduce it to the form $I$ or to a matrix with a zero row. Let the elementary row operations used be $E_1, \ldots, E_k$. Let $A$ be $m \times m$. For each