Math 121A – Homework 8

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Reading: Sections 3.3 and 3.4. In lecture, I present bits and snippets of those sections, as well as extra material about pivots. Remember that you’re responsible for all the material in the book, even the stuff I didn’t cover in lecture. In 3.3, ignore the section “An Application.”

Note: Remember that the lowest 2 homeworks/quizzes are dropped, so for some of you, this might be your last homework (yay!), but remember that all the homework problems are fair game for the final exam.

- **Section 3.3:** 1, 2(d), 4(b), 7(a), 8(a), 10 (Optional: AP)
- **Section 3.4:** 1, 2(f), 3, 5, 7, 10 (Optional: 9)

(TURN PAGE for AP)
Optional Additional Problem:

(a) Prove the following generalization of Theorem 3.9:

If $T : V \to W$ is linear and $w \in W$ is given, then the general solution $v \in V$ of the equation $T(v) = w$ is of the form $v = v_0 + v_p$, where $v_0$ is the general solution of $T(v) = 0$ and $v_p$ is a particular solution of $T(v) = w$.

**Hint:** Consider $v - v_p$

(b) Let $V = W = C^\infty(\mathbb{R})$ (infinitely differentiable functions from $\mathbb{R}$ to $\mathbb{R}$) and define

$$T(y) = y'' - 5y' + 6y$$

Show $T$ is linear

(c) (Requires Math 3D) Find the general solution of $T(y) = 0$

(d) (Requires Math 3D) Find a particular solution of $T(y) = e^x$

**Hint:** Undetermined coefficients or variation of parameters

(e) Use (a) to find the general solution of $y'' - 5y' + 6y = e^x$