MATH 121A – MIDTERM REVIEW SESSION

Problem 1: Suppose W_1 and W_2 are subspaces of a vector space V. Show that $W_1 \cup W_2$ is a subspace of V if and only if $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$.

Solution: Union of subspaces

Problem 2: Suppose W and Z are subspaces of a vector space V. We say $V = W \oplus Z$ if $V = W + Z$ and $W \cap Z = \{0\}$. Suppose $V = W \oplus Z$ and suppose $\{w_1, \cdots, w_m\}$ is a basis of W and $\{z_1, \cdots, z_k\}$ is a basis of Z. Show that $\{w_1, \cdots, w_m, z_1, \cdots, z_k\}$ is a basis of V

Solution: Direct Sums

Problem 3: Let $V = C^\infty(\mathbb{R})$ (the space of infinitely-differentiable functions from $\mathbb{R}$ to $\mathbb{R}$), and define $D : V \to V$ by $D(f) = f'$. Show that $\{D, D^2, D^3\}$ is linearly independent in $\mathcal{L}(V)$.

Solution: Derivative is linearly independent

Problem 4: Suppose V and W are vector spaces with $\dim(V) \geq \dim(W)$, and suppose Z is a subspace of W. Show that there exists a linear transformation $T : V \to W$ whose range is equal to Z.

Solution: Linear Transformation with a given range

Problem 5: Define $T : P_1(\mathbb{R}) \to \mathbb{R}^2$ by $T(a_0+a_1x) = (5a_0+2a_1, 2a_0+a_1)$. Find a formula for T^{-1}.

Solution: Calculate T^{-1}

Date: Friday, May 3, 2019.