MATH 121A – MIDTERM

Name: ________________________

Student ID: ________________________

Instructions: Welcome to your Midterm! You have 50 minutes to take this exam, for a total of 100 points. No books, notes, calculators, or cellphones are allowed. Remember that you are not only graded on your answer, but also on your work, so please write in complete sentences and explain your steps as much as you can. If you need to continue your work on the back of the page, clearly indicate so, or else your work will be discarded. **You will lose points if you don't sign the statement below**. May (the) 3^{rd} be with you!

Academic Honesty Statement: I hereby certify that the exam was taken by the person named and without any form of assistance and acknowledge that any form of cheating (no matter how small) results in an automatic F in the course, and will be further subject to disciplinary consequences, pursuant to section 102.1 of the UCI Student Code of Conduct.

Signature: _________________________________

1		10
2		30
3		30
4		30
Total		100

Date: Friday, May 3, 2019.

1. *(10 points)* Let $V = W = P_2(\mathbb{F})$ and define $T : V \to W$ by

$$T(p(x)) = p''(x) + p'(x) + 2p(0)$$

Find the matrix $[T]_\beta^\gamma$ of T, where $\gamma = \beta = \{1, x, x^2\}$

Note: In this problem, you do **NOT** need to show your work!

2. *(30 = 10 + 20 points)*

(a) State the Replacement Theorem

(b) Suppose S_1 and S_2 be finite subsets of a vector space V, with S_1 linearly independent, $Span(S_2) = V$, and suppose S_2 has the same number of elements as S_1. Show that S_1 is a basis of V.

3. *(30 points)* State and prove the Rank-Nullity Theorem (also known as the Dimension Theorem)

4. *(30 = 5 + 25 points)*

(a) Define: T is an isomorphism from V to W
Note: You do **NOT** need to define the terms that you're using in your definition

(b) Let V be the vector space of 2×2 matrices with trace 0. Find an explicit formula of an isomorphism from V to $\mathbb{F}^n$ (for some n of your choice) and show that it's an isomorphism.