MATH 121A – FINAL EXAM – STUDY GUIDE

PEYAM RYAN TABRIZIAN

General Info

The Final Exam takes place on **Tuesday, June 11** from 1:30 pm to 3:30 pm in our usual lecture room (PSCB140). Please bring your student ID to the exam room. I will also send out a seating chart the day before the exam.

This is the study guide for the exam. Please read it carefully, because it contains a lot of info about what's going to be on the exam and what I expect you to know or not to know. That said, remember that this study guide is just a *guide* and not a complete list. I've tried to make this list as complete as possible, but there are always things that I may have missed.

The final exam covers everything from section 1.2 to 5.2 inclusive. It is cumulative and should cover roughly one question per chapter. There will be at least one question from dual spaces (section 2.6), so make sure to study that section particularly well.

There will be roughly 8 problems on the exam. Just like for the midterm, the problems will be a mix definitions, proofs, homework problems (including the optional ones), and computational problems. There will also be a set of True/False questions. If you want to have an idea of the format of the exam, please look at the practice exam. The *best* way to prepare for the exam is to read the book and go over the homework problems. You don't need to study the quizzes or the discussion worksheets, since I didn't look at them at all.

Careful: I expect this final to be a bit harder than the midterm, so don't be fooled by the difficulty of the midterm. Also remember that you are graded not only on the correct answer, but also on the way you write out your answer. Don't be surprised if you get lots of points off if your proof is too short or if you skip too many details! If you have any doubts of whether you need to justify a part of the answer, do it! And write in complete sentences, not just mathematical symbols.

Date: Tuesday, June 11, 2019.

20 PROOFS YOU ABSOLUTELY NEED TO KNOW

Know the statements **and** the proofs of the following 18 theorems. You are *guaranteed* to have at least one of those 18 proofs on the exam.

Warning: Although I won't explicitly ask you to reprove theorems not on this list, you'll still need to know their statements and how to use them, so it's still worthwile to look at them because they use important techniques covered in this course.

Theorem	Name	YouTube Video
1.7 in 1.5	Intruder Theorem	Video
Cor 1 and 2(b)(c) in 1.6	Corollaries of Replacement Theorem	Video
2.3 in 2.1	Dim Thm/Rank-Nullity Theorem	Video
2.5 in 2.1	One-to-one is equivalent to onto	
2.6 in 2.1	Linear Extension Theorem	Video
2.19 in 2.4	Isomorphic iff same dimension	Video
Cor in 2.6	Every basis is a dual basis	Video
3.5 in 3.2	$Rank(A) = \dim(Col(A))$	Video
Cor 2a in 3.2	$Rank(A^T) = Rank(A)$	Video
3.9 in 3.3	Hom and part sol	Video
3.13 + Cor in 3.4	Row ops preserve sol	Video
4.3 in 4.2	Multilinearity of det	Video
Cor (215) in 4.2	Identical Rows	
4.5 in 4.2	Interchange rows	Video
4.8 in 4.3	$\det(A^T) = \det(A)$	Video
Pg. 4 in Lec 26	Char poly indep of basis	
5.5 in 5.2	Eigenvectors lin ind	Video
Pg. 5 in Lec 27	A diag iff $A = QDQ^{-1}$	Video

YOUTUBE PLAYLISTS

There are a bunch of videos on my YouTube channel, based on the concepts covered in this course. Check them out if you need help with a topic. The last video is a set of 111 T/F questions, which is based mostly on Math 3A stuff, but it should be relevant to our course too.

Chapter	YouTube Playlist
1	Vector Spaces
2	Linear Transformations and Matrices
2.6	Dual Spaces
3	Linear Equations
4	Determinants
5	Diagonalization
	111 True False Questions

SECTION 1.2: VECTOR SPACES

- You do **NOT** need to know the axioms of vector spaces (VS1)-(VS8), just know how to use them
- Look at Theorem 1.1 (Cancellation Law for Vector Addition), Corollary 1, Corollary 2, and Theorem 1.2(a) and (c). You don't need to memorize their proofs.
- Ignore the Additional Problem on HW 1
- Know how to show something is not a vector space (1.2.17-18)
- If you need more examples of vector spaces, check out this video: 25 Ex of VS

SECTION 1.3: SUBSPACES

- Know the definition of a subspace and the statement of Theorem 1.3 (but not its proof)
- Know how to show that something is or is not a subspace (for example, see 1.3.10) Also check out the examples in Lecture 2, as well as those videos: Subspace and Not Subspace.
- Know the statement and proof of Theorem 1.4 about intersection of subspaces
- Give an example showing the union of subspaces is not necessarily a subspace, also look at 1.3.19 and this video: Union of subspaces
- Know how to carry out a proof by induction, see 1.3.20

SECTION 1.4: LINEAR COMBINATIONS AND SYSTEMS OF LINEAR EQUATIONS

- Know the definition of linear combination
- Ignore Example 1 in the book
- For a review of row-reducton, check out Gaussian elimination
- Determine if a vector is a linear combination of other ones (Example 2 in the book, and 1.4.5)

- Know the definition of the span of a set and know (in theory) how to prove Theorem 1.5, also see Span is a subspace, although I won't explicitly ask you to show that.
- Know the definition of spanning set of V
- Figure out if vectors span a given space (Examples 3-5, and 1.5.10); remember that there is a much better technique in Chapter 3!
- Also check out 1.5.13 and 1.5.15

Section 1.5: Linear Dependence and Linear Independence

- Know the definition of linear dependence and linear independence
- Know how to show whether a set if linearly dependent or linearly independent. Again, feel free to use row-reduction for it. Check out Linear Independence if you want.
- Know the statement and Theorem 1.6 and its Corollary, but you don't need to memorize its proof
- Know the statement and proof of the "Intruder Theorem" (Theorem 1.7). Intuitively it's saying that if you add a vector $\mathbf{v}$ to a linearly independent set, and if that set becomes linearly dependent, then it's $\mathbf{v}$'s fault. See this video for the proof: Intruder Theorem
- Also check out 1.5.10, 1.5.14 and 1.5.15 (if you want), but ignore 1.5.20

Section 1.6: Bases and Dimension

- Know the definition of a basis and how to show something is a basis (like in 1.6.3)
- Know the statement and proof of Theorem 1.8
- Give examples of bases of vector spaces
- Find a basis of the span of a set, as in Example 6 or in 1.6.7, but remember there there is an easier way of doing this in Chapter 3.
- Know the statement, but not the proof of Theorem 1.9. You might also want to check out the proof I gave in Lecture 5, just as an example of how to use induction.
- Know the statement of the Replacement Theorem (Theorem 1.10). You do **NOT** need to know the proof of the replacement theorem. See this video for an explanation of the replacement theorem: Replacement Theorem.
- Know how to use the Replacement Theorem, in particular know the statements and proof of Corollary 1 and Corollary 2(b)(c) in the book. Also know how to do AP1 in Homework 3. You don't need to know the proof of Corollary 2(a), but know its statement. See the

above video for an explanation. Also remember the problem on the midterm about the replacement theorem.
- **Note:** Corollary 2(c) and the Corollary on page 51 are very important and will be used throughout the course
- Know the definition of dimension and explain why the definition makes sense. See this video for an overview: Dimension.
- Use the notion of dimension to your advantage, as in 1.6.4
- Find the dimension of a subspace, like in 1.6.14 or 1.6.16
- Give an example of an infinite-dimensional vector space, as in Infinite Dimensions
- Unless otherwise specified, do **NOT** assume your vector space is finite-dimensional
- Know the statement of Theorem 1.11, but not its proof.
- **IGNORE** the section on the Lagrange Interpolation Formula, but if you're curious, check out: Lagrange Interpolation Formula
- Don't forget about 1.6.29, see $\dim(V + W)$. You do **NOT** need to know the definition of sum or direct sums of subspaces; I would give you that definition if necessary. A good practice problem would be: Direct Sums
- Section 1.7 (Maximal Linearly Independent Subsets) will **NOT** be on the exam, but if you're curious how to show any vector space has a basis, check out: Maximal Linearly Independent Subsets

SECTION 2.1: LINEAR TRANSFORMATIONS, NULL SPACES, AND RANGES

- Know the definition of a linear transformation, and know the statement (but not the proof) of properties $1 - 4$ on page 65
- Know how to show that T is linear (Example 1 or Example 6) or not linear (2.1.9). Check out Linear Transformations
- Ignore Example 2
- Know the definition of $N(T)$ and $R(T)$, the statement and proof of Theorem 2.1 (although I won't explicitly ask you to prove it), the statement but not the proof of Theorem 2.2, and know how to find bases for $N(T)$ and $R(T)$ (Example 10). See also Nullspace is a Subspace.
- Know the definition of the rank and the nullity of T and know how to calculate them
- Know the statement and the proof of the Dimension Theorem/Rank-Nullity Theorem (Theorem 2.3, or see Rank-Nullity Theorem Proof), and know the statement and proof of Theorem 2.5, which is a cool

consequence of the Rank-Nullity Theorem, and check out Examples 11 and 12
- Know the definition of one-to-one and know the statement but not the proof of Theorem 2.4
- Know the statement and proof of Theorem 2.6, also known as the Linear Extension Theorem (I did this at the end of Lecture 8, see also Linear Extension Theorem). It's used everywhere in Linear Algebra, especially look at Problem 4 of the practice exam. Also remember the Corollary on page 73.
- 2.1.2, 2.1.6, and 2.1.15-16, 2.1.21, and AP2 on Homework 4 (see One to one iff Linearly Independent) are good review problems, as well as this video One-to-one and Onto and I highly recommend looking at 2.1.13 and 2.1.14(c)
- You don't need to know the definition of T-invariant (as in 2.1.28)
- Ignore problems 2.1.37 and 2.1.39, as well as AP2 on Homework 3 (but see Graphs of Linear Transformations) and AP1 on Homework 4
- Also check out the following videos (based on homework problems): Intersection of Range, Linear Transformation with a given range, and Derivative and Linear Independence

Section 2.2: The Matrix Representation of a Linear Transformation

- Know the definition of the coordinate vector of $\mathbf{x}$ relative to a basis β (page 80)
- Find the coordinates of $\mathbf{x}$ with respect to β. Again, totally ok to use row-reduction here. Also see Coordinates
- Know the definition of the matrix of T, $A = [T]_\beta^\gamma$ (bottom of page 80). Know both formulations, the one with $T(\mathbf{v_j})$ as a sum (right above the definition) and the one in terms of j-th column of A is $[T(\mathbf{v_j})]_\gamma$ (right below the definition)
- Know how to find the matrix of T relative to a basis (see 2.2.5, 2.2.10, Derivative in a Box, Matrix of a Matrix, Matrix with respect to a basis)
- Know the definition of sum and scalar multiplication of linear transformations (page 82) and know the statement of Theorem 2.7, but not its proof
- Know the definition of $\mathcal{L}(V, W)$ and $\mathcal{L}(V)$
- Know the statement, but not the proof of Theorem 2.8, as well as Example 5
- Also check out 2.2.13, 2.2.14, 2.2.16, see Pseudo Diagonalization

SECTION 2.3: COMPOSITION OF LINEAR TRANSFORMATIONS AND MATRIX MULTIPLICATION

- Know the definition of the composition UT of U and T and know the statement and the proof of Theorem 2.11 (the proof is on page 87), and the statements but not the proofs of Theorems 2.9 and 2.10 and the Corollary on page 89.
- Know the definition of AB, but this time you don't need to know how to derive it (page 87, also see Where Matrix Multiplication comes from, although I'm using slightly different notation in the video)
- Know how to calculate AB, see Matrix Multiplication, and remember that in general, $AB \neq BA$, see AB vs. BA
- Know the definition of A^T and know how to show $(AB)^T = B^T A^T$
- Know the properties in Theorem 2.12, but don't worry about the proof.
- For Theorem 2.13, you don't need to know the statement or the proof, but know how to use it. I clarified the statements a bit at the very end of Lecture 10.
- Know the statement but not the proof Theorem 2.14. You might want to look at the proof I gave at the beginning of Lecture 11. Also look at Example 3 to see how to use it.
- Know the definiton of L_A and the statements but not the proofs of Theorem 2.15
- Also know the statement but not the proof of Theorem 2.16; look at how cool it is!
- Ignore the section on Applications
- Also look at 2.3.9, 2.3.11, 2.3.12(a)(b), and 2.3.13
- Optional, but really cool: $A^2 = A$, $A^2 = O$, $A^2 = I$

SECTION 2.4: INVERTIBILITY AND ISOMORPHISMS

- Know the definitions of 'inverse of T' and 'T invertible'
- Know how to show that T is invertible (see 2.4.2)
- Know properties 1 and 2 on page 100, but you don't need how to prove them, and ignore property 3
- Know the statement but not the proof of Theorem 2.17. You can use either the book's proof of the proof I gave in Lecture 12
- Know the definition of 'A invertible' and A^{-1}
- Show that A^{-1} is unique (see statement at the bottom of page 100)
- Know the statement but not the proof of the Lemma on page 101
- Remember that if $\dim(V) = \dim(W) < \infty$, then one-to-one is equivalent to onto, that should be useful

- Know the statement and proof of Theorem 2.18, as well as the statements (but not the proofs) of Corollaries 1 and 2; see this video: Matrix of T-1
- Know the definition of isomorphism
- Know how to show that two vector spaces are isomorphic, both by directly writing down an isomorphism and showing that it's linear, one-to-one, and onto (2.4.14 and Isomorphism), or by counting the dimensions of V and W (see Theorem 2.19, or 2.4.3)
- Know the statement but not the proof of Theorem 2.19, see Isomorphism and Dimension
- Know the statement and proof of Theorem 2.20
- You don't need to know the definition of the standard representation (page 104), and you don't need to know the formula in the middle of page 105, but you need to know the statement and proof of Theorem 2.21 and to understand the diagram in Figure 2.2 and Example 7, see the following videos: 2 Miracles of Linear Algebra and Differentiate with Linear Algebra
- Know how to calculate T^{-1}, see this video for an example: Calculate T^{-1}
- Check out 2.4.4, 2.4.9, 2.4.10(a)(b)

Section 2.5: The Change of Coordinates Matrix

- This section is very badly written, so as long as you understand the notes from lecture and the homework problems, you're good to go!
- Know the definition of the change of coordinates matrix from β to γ and how to find it, see 2.5.2(b), 2.5.3(d). See this video for example: Change of coordinates
- Given the matrix of T with respect to β, find the matrix of T with respect to γ, as in 2.5.4, 2.5.5, 2.5.6(a), or this video: Change of matrix
- Find the formulas for reflection about a line and projection about a line. See lecture or Example 3 in the book, or the following videos: Reflection on a Line and Projection on a Line
- Know the definition of A similar to B and understand how it relates this section.

Section 2.6: Dual Spaces

- Remember that you are **guaranteed** to have a question about dual spaces on the exam!

- Know the definition of a dual space and linear functional, and know how to show that something is a linear functional (2.6.2), see Dual Spaces
- Know the definition of dual basis
- Given a basis β of V, find an explicit formula for vectors in the dual $\beta^{\star}$, see 2.6.3 and How to find a dual basis
- Know the statement but not the proof of Theorem 2.24. In particular, know the useful decomposition given in Theorem 2.24, see Dual Basis
- Know the definition of T^T and the statement but **not** the proof of Theorem 2.25, see Transpose Definition
- Know how to calculate T^T explicitly, as in Example 5 or 2.6.6Transpose Example
- Know the definition of $V^{\star\star}$ and the definition of $\hat{x}$, check out $V^{\star\star}$
- Know the statement by not the proof of Theorem 2.26
- Know the statement and the proof of the Corollary on page 123, see Every basis is a dual basis
- For additional practice, check out: 2.6.11, 2.6.13(a), 2.6.14, 2.6.15, 2.6.16, 2.6.19, 2.6.20, and if you're feeling adventurous, 2.6.10. They are based on the following videos: T^T one to one iff T onto, Annihilator of a subspace, $Rank(A^T) = Rank(A)$, Dual Lagrange Interpolation.
- Ignore the Additional Problem about Dirac Deltas on the homework

SECTION 3.1: ELEMENTARY MATRIX OPERATIONS AND ELEMENTARY MATRICES

- Know the three types of elementary matrices and their inverses, see Elementary Matrix
- Know how to write row-reduction as a product of elementary matrices
- Check out 3.1.9, it's very cute! See Unnecessary Operation?

SECTION 3.2: THE RANK OF A MATRIX AND MATRIX INVERSES

- Know the definition of rank of a linear transformation and the rank of a matrix
- Find the rank of a matrix, see 3.2.5
- Know the statement but not the proof of Theorem 3.4 and the Corollary
- Know the definition of the Column space and the proof that $Rank(A) = \dim(Col(A))$ (see lecture or Theorem 3.5), also see $Rank(A) = \dim(Col(A))$

- Know the statement but **not** the proof of the Fundamental Rank Theorem (Theorem 3.6), as well as the statement of Corollary 1, see Fundamental Rank Theorem
- Know the statement and the proof of Corollary 2(a), see $Rank(A^T) = Rank(A)$
- Know the statement but not the proof of Corollary 3, and know how to explicitly write an invertible matrix as a product of elementary matrices, like 3.2.7, also see Product of Elementary Matrices
- Know the statement but not the proof of Theorem 3.7
- Know how to check that A is invertible, see Check A is invertible
- Know how to calculate A^{-1} and understand why that algorithm works, see Calculate A^{-1}
- Know how to find a formula for T^{-1}, like in Example 7 or 3.2.6, see Calculate T^{-1}
- If you're interested in applied math, you can check out (purely optional): LU Decomposition

Sections 3.3 and 3.4: Sytems of Linear Equations

- Remember that in lecture, I presented some extra material about pivots and column spaces. You are responsible for both the material in lecture and in the book.
- Ignore the section "An Application" in 3.3
- Skip Theorem 3.8
- Know the statement and proof of Theorem 3.9, it's very important, see Homogeneous and Particular Solutions, and know the statement but not the proof of Theorem 3.10
- As a neat application, check out the AP on HW 8. I might ask you to prove part (a), but on the exam you're not required to know Math 3D
- Know the Rank Criterion (Theorem 3.11 or page 4 of Lecture 21) but not its proof, see Rank Criterion, and know how to use it, see Rank Criterion Consequences
- Use row-reduction to find if a vector is in the range of a linear transformation (Example 6 in 3.3 or 3.3.8)
- Know the theorem and the proof in Lecture 20 that row operations preserve the solutions (Theorem 3.13 and its Corollary, or page 2 in the notes of Lecture 20), also see Row operations preserve solutions
- Know the definition of pivot (page 2 in Lecture 20) and the definition of row echelon form (page 3 in Lecture 20).
- Use Gaussian Elimination to solve systems and to determine if a system has a solution. You are totally allowed to use the term pivots

and any facts you know about them (see for example 3.3.2, 3.3.7, 3.4.2 or Gaussian Elimination)

- You don't need to know the proof that row reduction can turn any matrix into REF (Theorem 3.14), but if you're curious, you can check out: Every matrix has a REF
- Know the definition of Reduced Row Echelon Form, but you don't need to know the proof of the theorem that any matrix can be turned in RREF
- Ignore Theorem 3.15
- Know the fact but not the proof that RREF is unique, but if you're curious, check out Uniqueness of RREF, it's a really beautiful proof!
- Given a matrix A, find $Rank(A)$, a basis for $Col(A)$, $dim(Nul(A))$ and a basis of $Nul(A)$, and understand *why* it works, it's really a fundamental idea in linear algebra (see Lecture 21 or Nul(A), Col(A), Rank(A))
- Know some of the applications of row-reduction, like extracting a basis from a spanning set, see Finding a basis or Example 3 or 3.4.7, extending a linearly independent set to a basis, see Basis extension or Example 4 or 3.4.10
- You **DON'T** need to memorize Theorem 3.16, but please look at the section Summary of RREF on pages 3 and 4 of Lecture 22, it contains some important facts about RREF
- Know how to reconstruct A given its RREF, see Reconstructing a Matrix from RREF or 3.4.5

SECTION 4.1: DETERMINANTS OF ORDER 2

- Ignore everything from page 204 on
- Hopefully that section is pretty straightforward! Know all the properties on pages 2-4 of Lecture 23. All the properties will be generalized in section 4.2, so in theory you can skip this.
- Know how to calculate the area of a parallelogram (4.1.4) and know the definition of orientation
- Check out 4.1.11, it's a cute exercise with a nice generalization (see Determinant Puzzle). If you're curious, check out Characterization of the Determinant for a generalization, but this is optional)

SECTION 4.2: DETERMINANTS OF ORDER n

- Know the definition of the determinant and how to calculate a determinant, see Determinants and Bomberman or Examples 1, 2, 3 or 4.2.5 for a refresher.

- Unless otherwise specified, you can evaluate the determinant along any row or column, but I might force you sometimes to just evaluate it along the first row (but in this case I'll clearly state it on the exam)
- Know the statement and proof of Theorem 4.3. The proof in the book is a little bit awkward, I highly recommend you to look at the proof on pages 1-2 of Lecture 24, as it's (hopefully) easier to understand. See also Multilinearity for a full proof. Remember that you also need to check the case $r = 1$ separately, as on HW 9
- Know the statements but not the proofs of the lemma at the bottom of page 213 and of Theorem 4.4, but see Expand along any row if you're curious
- Know the statement and the proof of the Corollary on page 215
- Understand and know how to prove the effects of row reduction on the determinant (pages 4-5 of Lecture 24), also see Theorem 4.5 and 4.6 and Determinants and Row Reduction
- Use row-reduction to evaluate determinants, see Example 5 and Determinants and Row Reduction Example, also see 4.2.3 and 4.2.21
- Know the statement but not the proof that if A is not invertible, then $\det(A) = 0$ (Corollary on page 217)
- Finally, check out 4.2.28 and 4.2.29

SECTION 4.3: PROPERTIES OF DETERMINANTS

- Know the statement but not the proof of Theorem 4.7, but understand how we use the fact that A is a product of elementary matrices, see $\det(AB) = \det(A)\det(B)$
- Know the statement but not the proof of the Corollary on page 223 (although this is not particularly hard to prove, so you might want to check it out anyway), see $\det(A^-1) = \frac{1}{\det(A)}$
- Know the statement and the proof of Theorem 4.8, see $\det(A^T) = \det(A)$. I might ask you to prove that if E is elementary, then $\det(E^T) = \det(E)$, see 4.2.28. Also understand how you can use this to evaluate the determinant along any column (bottom of page 3 in Lecture 25), although I won't explicitly ask you to prove that.
- Know the statement but not the proof of Cramer's Rule (Theorem 4.9) and know how to use it, see Example 1 or Cramer's Rule (if you're curious about the proof, see Cramer's Rule Proof
- Don't forget about 4.3.22 (see Vandermonde Determinant for a solution). If you want to be amazed, check out Vandermonde Determinant OMG Way
- Check out 4.3.24, but in that case I would tell you the answer and you'd have to prove it by induction, see Neat Determinant

- Check out Determinant of a Block Matrix
- You don't need to know things about volumes of parallelipipeds or determinants and volumes, although if you're curious, you can check out (optional) the following applications: Determinants and Volumes, Determinants and Inverses, Adjugate Matrix, Characterization of the determinant

SECTION 5.1: EIGENVALUES AND EIGENVECTORS

- **Note:** Most of the things in the book and in lecture apply both to linear transformations and to matrices. You need to know how to do everything for both, like finding eigenvalues and eigenvectors.
- Find the eigenvalues and eigenvectors of a matrix and of a linear transformation, see Eigenvalues and Eigenvectors and Abstract eigenvalues and eigenvectors and 5.1.3, 5.1.4
- Check out the following exotic example of an eigenvalue: Oh shift !
- Know how to show that λ is an eigenvalue iff $\det(A - \lambda I) = 0$ (Theorem 5.2) and that v is an eigenvector of T iff $v \in N(T - \lambda I)$ (Theorem 5.4, see page 5 of Lecture 26), although I won't explicitly ask about it
- Know the definition of a characteristic polynomial of a linear transformation and to show that it is independent of the basis β (see 5.1.12 or pages 4-5 of Lecture 26). Know how to find the characteristic polynomial of a matrix and of a linear transformation and how to find the eigenvalues that way (Examples 4 and 5)
- Know how to show that v is an eigenvector of T if and only if $[v]_\beta$ is an eigenvector of A (page 1 on Lecture 27), although I won't explicitly ask to prove this. This is a very important fact that allows us to find eigenvectors of a linear transformation, as in Example 7
- Know the definition of diagonalizable (both for linear transformations and for matrices) and know the statement (but not the proof) that T (or A) is diagonalizable if and only if there is a basis β consisting of eigenvectors of T (or A)
- You don't need to know the geometric interpretation of eigenvectors (page 4 of Lecture 27), but it is very interesting and useful, check out this video if you're interested: Eigenvectors and Geometry
- Know how to show that A is diagonalizable if and only if $A = QDQ^{-1}$ for some D diagonal and Q invertible (page 5 of Lecture 27)
- You may also want to check out the Legend of Zelda analogy of diagonalization, see Diagonalization and Legend of Zelda

- Also check out 5.1.8, 5.1.11, 5.1.12, 5.1.14, 5.1.15, but ignore the Additional Problem on HW 10

SECTION 5.2: DIAGONALIZABILITY

- Know the definition of splitting, and know the statement (but not the proof) that if T is diagonalizable, then $f(t)$ must split (Theorem 5.6), as well as the Test 1 for diagonalizability (if f doesn't split, then T is not diagonalizable)
- Know the statement and the proof of Theorem 5.5 and its Corollary, the Test 2 for diagonalizability (if A has n distinct eigenvalues, then A is diagonalizable), see Eigenvectors and Linear Independence
- Know an example of a nondiagonalizable matrix (or linear transformation), see Not diagonalizable, and you might also want to check out Derivative not diagonalizable
- Beware that a $n \times n$ matrix with only one eigenvalue could or could not be diagonalizable
- Know the definition of algebraic multiplicity (page 263) and eigenspace (page 264)
- Know the statement but not the proof of Theorem 5.7
- Ignore the Lemma and Theorem 5.8 on page 267
- Know the statement but not the proof of the Ultimate Test 3 of diagonalizbility (Theorem 5.9(a) on page 268, see Ultimate Diagonalization Test and Ultimate Diagonalization Test Part 2), also know the statement but not the proof of Theorem 5.9(b).
- Use the 3 tests above show if a matrix or a linear transformation is diagonalizable or not, see Examples 5 and 6. You're welcome to use pivots and free variables instead of rank. In lecture I just did the case for matrices, but you're responsible for both the matrix case and the linear transformation case.
- Use diagonalization to find D diagonal and Q invertible with $A = QDQ^{-1}$, like in 5.2.2
- Use diagonalization to find a basis for which the matrix of T is diagonal, like in 5.2.3
- Use diagonalization to find A^n for any n. You're not responsible for calculating $\sqrt{A}$ (but check out Square root of a matrix if you're curious), see also 5.2.7
- Ignore the sections on System of Differential Equations and on Direct Sums; I would tell you the definition of direct sum if necessary, but check out Diagonalization and Direct Sums if you want
- Also check out 5.2.8, 5.2.11, 5.2.12