MATH 112A – MIDTERM

Name: _____

Student ID: _____

Instructions: Welcome to your Midterm! You have 50 minutes to take this exam, for a total of 100 points. **You will lose 1 point if you don't fill out all the information on this page**. No books, notes, calculators, or cellphones are allowed. Remember that you are not only graded on your answer, but also on your work. If you need to continue your work on the back of the page, clearly indicate so, or else your work will be discarded. And remember, the PDE doesn't solve itself :)

Academic Honesty Statement: I hereby certify that the exam was taken by the person named and without any form of assistance and acknowledge that any form of cheating (no matter how small) results in an automatic F in the course, and will be further subject to disciplinary consequences, pursuant to section 102.1 of the UCI Student Code of Conduct.

Signature: _____

1		25
2		25
3		25
4		25
Total		100

1. *(25 points)* Find the general solution of the following PDE

$$\left(y^2\right) x \, u_x + \left(x^3\right) y \, u_y = 0$$

2. *(25 points)* Solve

$$\begin{cases} u_{xx} - 3u_{xt} - 4u_{tt} = 0 \\ \quad u(x,0) = e^x \\ \quad u_t(x,0) = \cos(x) \end{cases}$$

Note: You may assume and you do **NOT** (!!!) need to show that the general solution of the above PDE is

$$u(x,t) = F(4x + t) + G(x - t)$$

Also, please don't waste your time at the end simplifying your final answer, but make sure it doesn't involve any constants.

3. *(25 points)* Suppose u and v both solve the heat equation $u_t = ku_{xx}$
   for $0 \leq x \leq l$ and $0 \leq t \leq T$ and that

$$u(x, 0) \leq v(x, 0)$$
$$u(0, t) \leq v(0, t)$$
$$u(l, t) \leq v(l, t)$$

   Show $u(x, t) \leq v(x, t)$ for all x and t

4. *(25 points)* Suppose u solves the following heat-like PDE for $0 < x < l$

$$\begin{cases} u_t = ku_{xx} - u^3 \\ u(0,t) = u(l,t) \\ u_x(0,t) = u_x(l,t) \\ u(x,0) = 0 \end{cases}$$

Use energy methods to show that $u(x,t) = 0$ for all x and t

Hint: Multiply your equation by u and integrate. Careful about the endpoints in the integration.

Note: Here u^3 means $(u(x,t))^3$ I know this is a PDE you haven't seen before, but the same techniques work here, so just trust yourself and remember that I believe in you :)