

## MATH 3A – APPLICATIONS OF DIAGONALIZATION

Here are some extra suggested problems related to the applications of diagonalization.

**Problem 1:** Let  $A$  be the matrix

$$A = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

(a) Find  $D$  and  $P$  such that  $A = PDP^{-1}$

(b) Calculate  $\sqrt{A}$

**Hint:**  $\sqrt{A} = A^{\frac{1}{2}}$ . Also, you can find solutions at the end of the  $\mathcal{B}$  matrix lecture notes (Lecture 20), as well as under the following link: [Matrix Square root](#)

(c) Calculate  $e^A$

**Hint:** Same thing, but replace  $\sqrt{\cdot}$  by  $e$ . You can find a similar solution on [Matrix Exponential](#)

(d) Calculate  $\cos(A)$

**Hint:** Same thing, but replace  $e$  with  $\cos$ . You can find a similar solution on [cos of matrix](#)

Of course, you can calculate other fun things, like  $\ln(A)$ ,  $\tan^{-1}(A)$ , basically whatever you want :)

**Problem 2:** Suppose  $F_n$  satisfies  $F_{n+1} = 5F_n - 6F_{n-1}$  with  $F_0 = 2$  and  $F_1 = 5$ . Find a formula for  $F_n$ . I believe the answer is  $F_n = 2^n + 3^n$ .

**Problems 3 and 4:** Check out Problem 10 on the Peyam 1 and Peyam 2 Final Exams on my webpage (which were the finals I gave out last winter)