# Math 112A – Homework 5

Peyam Tabrizian

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**Reading:** Sections 3.1 and 3.2. I will probably have started with 4.1 already, but you won't be responsible for it until Homework 6. In section 3.2, do **NOT** read the section on 'The Finite Interval,' we won't cover that. Also, do **NOT** memorize all the formulas for solutions like (6) in 3.1, but definitely be able to derive them!

**Note:** In some of the problems, by 'Solve,' the book means write the answer in terms of the error function

$$\mathcal{E}rf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy$$

- **Section 3.1:** 1 (see hint), 2 (see hint), 3, AP1, AP2, AP3
- **Section 3.2:** 1 (what they mean is $u_x(0,t) = 0$), 5 (ignore the part about singularity), AP4

**Additional Problems:**

**AP 1:** Use your answer in Problem 3 in 3.1 to derive the solution formula for the following half-line Neumann problem for $x > 0$. Do **NOT** simplify the resulting expression, but please write it in terms of $\phi$.

$$\begin{cases} w_t &= k w_{xx} \\ w_x(0,t) &= 3 \\ w(x,0) &= \phi(x) \end{cases}$$

**Hint:** Let $v(x,t) = w(x,t) - ?$ where this time ? is a function whose $x-$derivative is 3.

**AP 2:** The following problem has nothing to do with PDEs, but illustrates something fun about even and odd functions

(a) Show that any function $f(x)$ can be written as a sum of an even function and an odd function.[1]

**Hint:**

$$f(x) = \left(\frac{f(x) + f(-x)}{2}\right) + \left(\frac{f(x) - f(-x)}{2}\right)$$

(b) What do you get in the case $f(x) = e^x$? Do those functions have a special name? We'll see them again in Chapter 6.

**AP 3:** Let $f$ be any function defined on $(0, \pi)$.

(a) Express the following quantity (called the Fourier cosine coefficient, see Chapter 5) in terms of $\int_0^\pi f(x)\cos(nx)dx$

$$\frac{1}{\pi}\int_{-\pi}^{\pi} f_{even}(x)\cos(nx)dx$$

Here $f_{even}$ is the even extension of $f$ on the interval $(-\pi, \pi)$

(b) Same as $(a)$, but replace $\cos$ with $\sin$ and $f_{even}$ with $f_{odd}$.

**AP 4:**

(a) Solve the following wave equation on the half-line $x > 0$. Do **NOT** simplify the resulting expression, but please write everything in terms of $\phi$ and $\psi$.

$$\begin{cases} w_{tt} = c^2 w_{xx} \\ w(0,t) = 1 \\ w(x,0) = \phi(x) \\ w_t(x,0) = \psi(x) \end{cases}$$

[1] In fact this decomposition is unique, but you don't have to show that

(b) Use the result of Problem 1 in 3.2 to solve the following Neumann problem on the half-line $x > 0$. Do **NOT** simplify the resulting expression, but please write everything in terms of $\phi$ and $\psi$.

$$\begin{cases} w_{tt} = c^2 w_{xx} \\ w_x(0,t) = 3 \\ w(x,0) = \phi(x) \\ w_t(x,0) = \psi(x) \end{cases}$$

**Hint for 1 in 3.1:** I know, this problem is a bit long ☹

My suggestion is, instead of starting with the definition of $S$, first write everything in terms of $S(x-y)$ and $S(x+y)$ then use the u-sub $p = x - y$ (for the first integral), and $q = x + y$ (for the second one), and *then* only use the explicit formula of $S$ and do the completing the square trick. Also remember that by solving, the book means write the answer in terms of $\mathcal{E}$rf(x).

**Hint for 2 in 3.1:** There isn't much you need to do, because you can use the result of Example 1 (with suitable modifications)