Math 112A – Homework 6

Peyam Tabrizian

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Reading: Sections 4.1 and 4.2, including the bit at the end of the Lecture 20 notes about Inhomogeneous boundary conditions.

Note: The book and I use slightly different conventions: I use $\lambda$ whereas the book uses $-\lambda$. Both answers are acceptable, and for this course it won’t really matter since in the end our solutions will be the same.

Beware: Do NOT memorize the formulas for the solutions; on the exam you will need to derive everything. And remember that on the exam you have to show ALL your work. You will lose points on the exam if you skip steps (like not doing the 3 cases for solving for $X$).

- Section 4.1: 2, 3, 4, 6, AP1, (Optional: AP2)
- Section 4.2: 1, 2, 3, 4

(Mandatory) AP1:

(a) Carefully (= showing all your steps, including the 3 cases of solving $X'' = \lambda X$), find the solution of (here $0 < x < 1$)

$$
\begin{align*}
\left\{ \begin{array}{l}
 u_t &= ku_{xx} \\
 u(0, t) &= 0 \\
 u(1, t) &= 0 \\
 u(x, 0) &= 3 \sin(2\pi x)
\end{array} \right.
\end{align*}
$$
Note: Notice that here you can actually solve for your constants by comparing coefficients, see end of the Lecture 18 notes.

(b) Use (a) to solve

\[
\begin{align*}
&u_t = ku_{xx} \\
&u(0, t) = 5 \\
&u(1, t) = 5 \\
&u(x, 0) = 3\sin(2\pi x) + 5
\end{align*}
\]

**Hint:** Check out the end of the Lecture 20 notes to see how to do that (in case I don’t get to it)

(c) Use (a) to solve

\[
\begin{align*}
&u_t = ku_{xx} \\
&u(0, t) = 3 \\
&u(1, t) = 5 \\
&u(x, 0) = 3\sin(2\pi x) + (2x + 3)
\end{align*}
\]

**Hint:** Same hint as (b).

**Optional** AP2: Find a solution of the following PDE using separation of variables:

\[
(u_t)^2 (u_{tt}) + 2 (u_x) (u_t) (u_{xt}) + (u_x)^2 (u_{xx}) = 0
\]

**Hint:** Use \(u(x, t) = X(x) + T(t)\). Here there is actually a solution for any \(\lambda\). Remember that we only need to find one solution. To simplify your work, use \(\lambda = \frac{1}{3}\) (or \(-\frac{1}{3}\) if you’re using the book’s convention) and set any other constants you find equal to 0.