Math 112A – Homework 6

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Reading: Sections 4.1 and 4.2, including the bit at the end of the Lecture 20 notes about Inhomogeneous boundary conditions.

Note: The book and I use slightly different conventions: I use λ whereas the book uses $-\lambda$. Both answers are acceptable, and for this course it won't really matter since in the end our solutions will be the same.

BEWARE: Do **NOT** memorize the formulas for the solutions; on the exam you will need to derive **everything**. And remember that on the exam you have to show **ALL** your work. You **will** lose points on the exam if you skip steps (like not doing the 3 cases for solving for X).

- **Section 4.1:** 2, 3, 4, 6, AP1, (Optional: AP2)
- **Section 4.2:** 1, 2, 3, 4

(Mandatory) AP1:

(a) **Carefully** (= showing *all* your steps, including the 3 cases of solving $X'' = \lambda X$), find the solution of (here $0 < x < 1$)

$$\begin{cases} u_t &= k u_{xx} \\ u(0,t) &= 0 \\ u(1,t) &= 0 \\ u(x,0) &= 3sin(2\pi x) \end{cases}$$

Note: Notice that here you can actually solve for your constants by comparing coefficients, see end of the Lecture 18 notes.

(b) Use (a) to solve

$$\begin{cases} u_t = & ku_{xx} \\ u(0,t) = & 5 \\ u(1,t) = & 5 \\ u(x,0) = & 3sin(2\pi x) + 5 \end{cases}$$

Hint: Check out the end of the Lecture 20 notes to see how to do that (in case I don't get to it)

(c) Use (a) to solve

$$\begin{cases} u_t = & ku_{xx} \\ u(0,t) = & 3 \\ u(1,t) = & 5 \\ u(x,0) = & 3sin(2\pi x) + (2x+3) \end{cases}$$

Hint: Same hint as (b).

(Optional) AP2: Find a solution of the following PDE using separation of variables:

$$(u_t)^2 (u_{tt}) + 2 (u_x) (u_t) (u_{xt}) + (u_x)^2 (u_{xx}) = 0$$

Hint: Use $u(x,t) = X(x) + T(t)$. Here there is actually a solution for any λ. Remember that we only need to find one solution. To simplify your work, use $\lambda = \frac{1}{3}$ (or $-\frac{1}{3}$ if you're using the book's convention) and set any other constants you find equal to 0.