Math 112A – Homework 7

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Reading: Sections 5.1 (and 5.2 & 5.4). Please do **NOT** read 5.2 and 5.4; they're impossible to read. As long as you understand what I covered in lecture, then you're fine.

Note: The book and I use slightly different conventions: I define A_0 as $\frac{1}{l}$ times the integral but then just use A_0 in the Fourier series. The book defines A_0 as $\frac{2}{l}$ times the integral but then uses $\frac{A_0}{2}$ in the Fourier series. Both conventions give you the same answer at the end, so feel free to choose whichever one you prefer. In any case, don't forget to divide by 2.

- **Section 5.1:** 2, AP1, AP2, AP3, (Optional: 5)
- **Section 5.2:** 11 (see end of Lec 22 notes on my webpage)
- **Section 5.4:** 5(b)(d), 6, 12, 13, (Optional: 14, AP4)

Note: In some of the problems, the following trig identities are useful (do **NOT** memorize them):

(1)
$$\sin(A)\sin(B) = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$$

(2)
$$\cos(A)\cos(B) = \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(A-B)$$

(3)
$$\sin(A)\cos(B) = \frac{1}{2}\sin(A+B) + \frac{1}{2}\sin(A-B)$$

Additional Problem 1: Here is another reason why 2 appears in the Fourier sine (and cosine) series:

Suppose f is a function on $(0,\pi)$. Calculate the coefficients A_m and B_m of the **FULL** Fourier series of the oddification f_{odd} on $(-\pi,\pi)$ and write your answer using just f instead of f_{odd} (just like you did in AP3(b) in HW 5)

(Notice that this expansion is valid on $(0,\pi)$, and $f_{odd} = f$ on that interval, so this gives us the legit sine series of f)

Additional Problem 2: **Carefully** (= showing all your steps, including the 3 cases) solve the following heat equation with Neumann boundary conditions:

$$\begin{cases} u_t &= k u_{xx} \\ u_x(0,t) &= 0 \\ u_x(\pi,t) &= 0 \\ u(x,0) &= x^2 \end{cases}$$

Note: I have updated the Lecture 20 notes with the heat equation with Neumann boundary conditions, so feel free to check it out.

Additional Problem 3:

(a) Use the trig identities above to show that if $m \neq n$

$$\int_0^\pi \sin(mx) sin(nx) dx = 0$$

This is the key ingredient needed to show that $\{\sin(mx) \mid m = 1, 2, \cdots\}$ is orthogonal.

(b) Show that

$$\int_0^\pi \sin^2(mx) dx = \frac{\pi}{2}$$

Hint: Use $\sin^2(mx) = \frac{1-\cos(2mx)}{2}$

Optional Additional Problem 4: Apply Parseval's identity to the result in 11 in section 5.2 (with $l = \pi$) to find

$$\sum_{n=1}^{\infty} \frac{1}{n^2+1}$$

Note: Solution