Math 112A – Homework 8

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Congratulations for making it so far, this is your final homework assignment! I am so proud of you ☺

Reading: Sections 6.1 and 6.2. Do **NOT** read 6.3; all you need to know is the statement of the mean-value property (center of page 169). In 6.1, know the statement of the maximum principle (ignore its proof) and know how to derive uniqueness from it. In 6.2, know invariance in 2 dimensions, but ignore invariance in 3 dimensions **except** how to derive your solution **given** equation (6). In section 6.2, ignore Example 2; also I won't ask you to calculate double Fourier series, but in theory, you should be able to derive the coefficients. You will continue with the rest of chapter 6 in 112B.

Beware: For spherical coordinates, the book uses r instead of ρ and switches θ and ϕ. Since we'll set things equal to 0, it won't really matter.

Important: To simplify your task, for this HW **ONLY**, in 6.2 you don't need to show the 3 cases, but on the exam you'll definitely have to! Also, there are more hints below, after the additional problems.

- **Section 6.1:** 2, 5, 7, 11 (ignore the question "Also show...") AP1, AP2
- **Section 6.2:** 3, 4, 7(a)
- **Section 6.3:** 1

Additional Problem 1: Here is a more natural way of deriving the solutions of $\Delta u = 0$. Here we illustrate the case $n = 3$, but the proof is valid for any n.

Suppose $u = u(x, y, z)$ satisfies $u_{xx} + u_{yy} + u_{zz} = 0$ and suppose $u = v(\sqrt{x^2 + y^2 + z^2})$ for some $v = v(r)$.

Find the ODE that v satisfies, and solve it, and finally solve for u.

Hint: Use the Chen Lu to calculate u_{xx} in terms of v and then just guess the formulas for u_{yy} and u_{zz}.

Additional Problem 2: We say $u = u(x, y, z)$ is **subharmonic** if $-\Delta u \leq 0$.

(a) Suppose u is harmonic, $f'' \geq 0$, and let $v = f(u)$. Show v is subharmonic.

(b) Suppose u is harmonic and let $w = (u_x)^2 + (u_y)^2 + (u_z)^2$. Show w is subharmonic

Hints to the book problems:

6.1.2 Don't do it from scratch; use the Laplacian in spherical coordinates (equation (6)) and set all the θ and ϕ terms to 0. It's also useful to first calculate v_r and $v_r r$.

6.1.5 Use the Laplacian in polar coordinates (equation (5)) and set all the θ terms to 0

6.1.7 Use the Laplacian in spherical coordinates (equation (6)) and set all the θ and ϕ terms to 0.

6.2.4 First solve $v_{xx} + v_{yy} = 0$ with

$$v(x, 0) = x, v(x, 1) = 0, v_x(0, y) = 0, v_x(1, y) = 0$$

and then $w_{xx} + w_{yy} = 0$ with

$$w(x, 0) = 0, w(x, 1) = 0, w_x(0, y) = 0, w_x(1, y) = y^2$$

and use $u = v + w$.

6.2.7 In this problem, it's (exceptionally) useful to not write your solutions in terms of cosh and sinh, but instead to leave it in terms of exponentials. Notice that terms with e^{cy} with $c > 0$ don't go to 0 as $y \to \infty$.

6.3.1 (a) Maximum principle, (b) mean-value formula (center of page 169).