MATH 3A – MIDTERM

Name: ________________________

Student ID: ________________________

Instructions: Welcome to your Midterm! You have 50 minutes to take this exam, for a total of 100 points. No books, notes, calculators, or cellphones are allowed. **You will lose 1 point if you don't fill out all the information on this page**. Remember that you are not only graded on your final answer, but also on your work. If you need to continue your work on the back of the page, clearly indicate so, or else it will be discarded. And row-reduce it once more time! :)

Academic Honesty Statement: I hereby certify that the exam was taken by the person named and without any form of assistance and acknowledge that any form of cheating (no matter how small) results in an automatic F in the course, and will be further subject to disciplinary consequences, pursuant to section 102.1 of the UCI Student Code of Conduct.

Signature: _________________________________

1		10
2		20
3		25
4		20
5		10
6		15
Total		100

Date: Friday, November 1, 2019.

1. *(10 points, 2 points each)* Label each statement as **TRUE** or **FALSE**. Each correct answer will get 2 points and each incorrect/illegible one will get 0 points. In this question, you do **NOT** have to justify your answer.

   (a) If A is a 4×7 matrix, then $Nul(A)$ is at least 3 dimensional.

   (b) If $A\mathbf{x} = \mathbf{0}$ has a nonzero solution, then $A\mathbf{x} = \mathbf{b}$ must be inconsistent for some $\mathbf{b}$.

   (c) If $T : \mathbb{R}^3 \to \mathbb{R}^3$ is an onto linear transformation, then T must also be one-to-one.

   (d) If I is the 3×3 identity matrix, then $\det(3I) = 3\det(I)$.

   (e) If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then $Nul(A) = Span\left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

2. *(20 points)* Find the inverse of A, or say it does not exist.

$$A = \begin{bmatrix} -5 & -1 & 7 \\ 3 & 1 & -2 \\ 1 & 0 & -2 \end{bmatrix}$$

3. *(25 = 10 + 3 + 6 + 3 + 3 points)* For the following matrix A, find:

(a) A basis for $Nul(A)$
(b) $\dim(Nul(A))$
(c) A basis for $Col(A)$
(d) $Rank(A)$
(e) State the Rank-Nullity Theorem

$$A = \begin{bmatrix} 2 & -6 & 12 & 4 & 40 \\ -2 & 6 & -11 & -8 & -45 \\ 4 & -12 & 24 & 10 & 84 \\ -2 & 6 & -12 & -4 & -40 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 6 & 2 & 20 \\ 0 & 0 & 1 & -4 & -5 \\ 0 & 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

4. *(20 points, 10 points each)* Find all the values of h (if any) for which

   (a) The linear transformation $T(\mathbf{x}) = A\mathbf{x}$ is one-to-one

   (b) The columns of A span $\mathbb{R}^3$

   Briefly justify your answers

$$A = \begin{bmatrix} 1 & -3 & -2 & 2 \\ 2 & -6 & -3 & 7 \\ 3 & -9 & -6 & h \end{bmatrix}$$

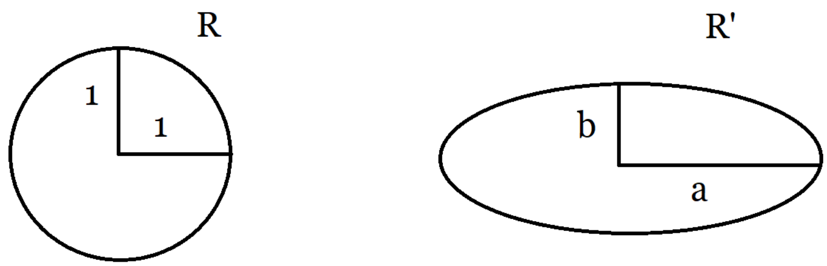
5. *(10 points, 5 points each)* Label each statement as **TRUE** or **FALSE**. In this question, you **have** to justify your answer.

   (a) If Q is square and $Q^T Q = I$, then $\det(Q) = \pm 1$

   (b) The matrix of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ which reflects points about the $y-$axis is

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

6. *(15 = 7 + 8 points)* Let R' be the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1$ in $\mathbb{R}^2$ (where a and b are positive) and R be the unit disk $x^2 + y^2 \leq 1$, as in the following picture:



(a) Find the matrix A of the linear transformation T that sends R to R', that is such that $T(R) = R'$.

(b) Using the formula of determinants in terms of areas, calculate the area of R'. No integrals allowed here!