LECTURE 29: FINAL EXAM REVIEW (III)

Monday, December 2, 2019 7:31 PM

There's a saying in German that says: "Everything has an end, except for a sausage, which has two," and with this I'd like to welcome you to the last lecture of Math 3A, and the third part of our final exam review session.

Today: All about True/False questions!

Note: There's a video on YouTube with 111 True/False questions, in case you need more practice.

(let's do something fun: For each question, I'll say the corresponding number in the world's most popular language)

(a) If A is invertible, then A is diagonalizable

(Mandarin: 1 = Yi)

FALSE

Example: $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ (prime example of a nondiagonalizable matrix)

Invertible (since det(A) is nonzero), but not diagonalizable (1 is an eigenvalue, but only one eigenvector, $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$)

(b) If A is diagonalizable, then A is invertible

(Spanish: 2 = Dos)

FALSE

Example: $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Diagonalizable (it's diagonal!), but not invertible

(c) If Q is orthogonal, then $(Qx) \cdot (Qy) = x \cdot y$

(English: 3 = Three)

TRUE

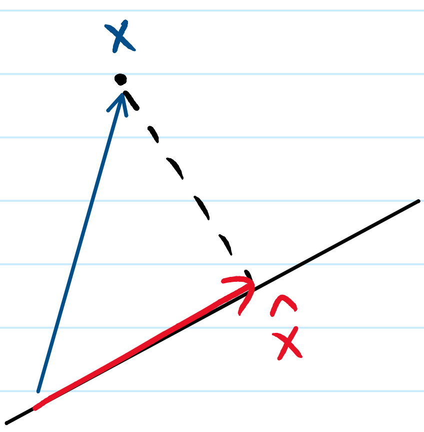
$(Qx) \cdot (Qy) = (Qx)^T (Qy) = x^T Q^T Q y = x^T I y = x^T y = x \cdot y$

(Interpretation: Orthogonal matrices not only preserve lengths, but angles as well)

(d) $\hat{x}$ is orthogonal to x

(Bengali: 4 = Thar)

FALSE



Not perpendicular!

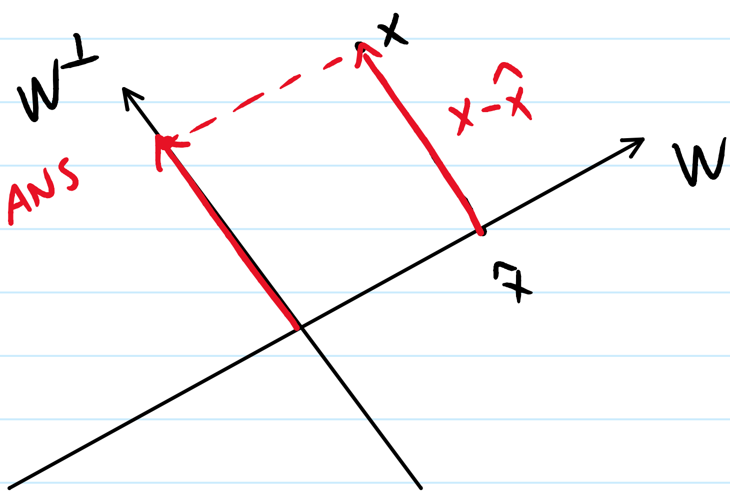
(What IS true is that $x - \hat{x}$ is orthogonal to $\hat{x}$)

(e) The OP of x on $W^{\perp}$ is $x - \hat{x}$

(Hindi: 5 = Panj)

TRUE

In all those problems, a picture helps



(f) If u and v are (nonzero) eigenvectors of A corresponding to λ and μ respectively, with $\lambda \neq \mu$, then {u,v} is LI

(Portuguese: 6 = Seis)

TRUE

Suppose $a u + b v = 0$ $(*)$ (show $a = b = 0$)

Know: $Au = \lambda u$, $Av = \mu v$

1) Apply A to $(*)$:

$A(au + bv) = A0$

$a\,Au + b\,Av = 0$

$a\,\lambda\,u + b\,\mu\,v = 0$

2) Hint: Multiply ($*$) by λ

$\lambda\,(au + bv) = \lambda\,0$

$a\,\lambda u + b\,\lambda\,v = 0$

3) $a\lambda\,u + b\mu\,v = 0$

$a\,\lambda\,u + b\,\lambda\,v = 0$

Subtract: $\cancel{a\lambda u} + b\mu\,v - \cancel{a\lambda u} - b\lambda v = 0$

$b\mu\,v - b\lambda\,v = 0$

$b\,\underbrace{(\cancel{\mu - \lambda})}_{\neq 0}\,v = 0$

$b\,v = 0$

$b = 0$ (since $v \neq 0$)

Finally, $au + bv = 0 \Rightarrow au + \underset{b}{\underset{\uparrow}{0}}v = 0 \Rightarrow au = 0 \Rightarrow a = 0$

So $a = b = 0$

(g) If A is similar to I, then $A = I$

(Russian: 7 = Siem)

TRUE

Know: $A = PIP^{-1} = PP^{-1} = I$

(h) The matrix $B = PDP^T$ is symmetric (here D is diagonal)

(Japanese: 8 = Hadshee)

TRUE

Show $B^T = B$

But $B^T = (PDP^T)^T = (P^T)^T D^T P^T = P D^T P^T = PDP^T$ (since D is diagonal) $= B$

(i) If A is symmetric, and u and v are eigenvectors of A corresponding to λ and μ with $\lambda \neq \mu$, then u and v are orthogonal

(German: 9 = Neun)

TRUE

Hint: Consider $(Au) \cdot v$

On the one hand, $(Au) \cdot v = (\lambda u) \cdot v = \lambda\, u \cdot v$

On the other hand:

$(Au) \cdot v = (Au)^T v = u^T A^T v = u^T A v$ (since A is symmetric)
$= u^T \mu v = \mu u^T v = \mu\, u \cdot v$

Hence $\lambda\, u \cdot v = \mu\, u \cdot v \Rightarrow \underbrace{(\lambda - \mu)}_{\neq 0} u \cdot v = 0 \Rightarrow u \cdot v = 0$

(j) $Ax = b$ always has exactly one least-squares solution

(Cantonese: 10 = Tsaa)

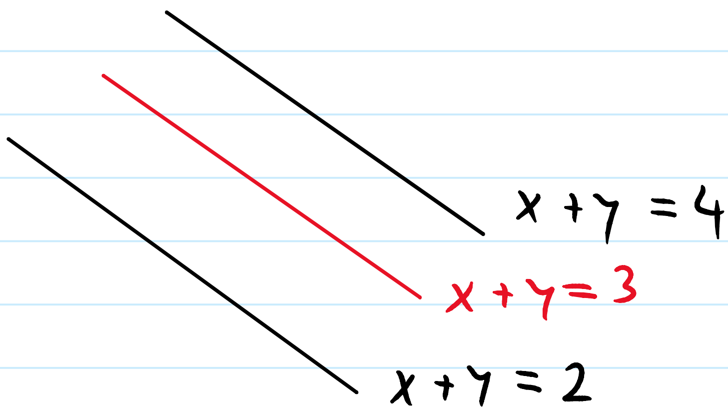
FALSE

Example:

$$\begin{cases} x + y = 2 \\ x + y = 4 \end{cases}$$

This has infinitely many L-S solutions, all on the line $x + y = 3$ (see HW)

Picture:



(k) If $T: R^n \to R^n$ is one-to-one, then T is also onto R^n

(Javanese: 11 = Sewelas)

TRUE

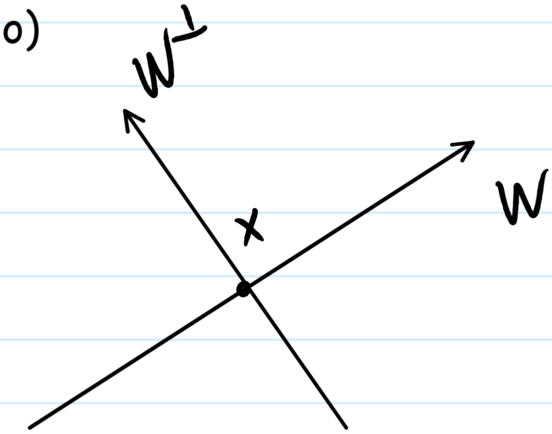
T 1-1 => A is invertible (by IMT, since A is nxn)
=> T is onto R^n (again by IMT)

(l) If x is in W and $W^\perp$, then x = 0

(Korean: 12 = 12 Yeldo)

TRUE

Picture:



Since x is in $W^{\perp}$, $x \cdot w = 0$ for all w in W
Let w = x (which is in W), then
$x \cdot X = 0$
=> x = 0

(m) Any system of 2 equations in 3 unknowns must has infinitely many solutions

(French: 13 = Treize)

FALSE

$$\begin{cases} x + y - z = 2 \\ x + y - z = 3 \end{cases}$$

(All we can say is: Ax = 0 must have infinitely many solutions)

(n) Any system of 3 equations in 2 unknowns must be inconsistent

(Vietnamese: 14 = Muy Bohn)

FALSE

$$\begin{cases} x + y = 2 \\ 2x + 2y = 4 \\ 3x + 3y = 6 \end{cases}$$

(All we can say: There is some b with $Ax = b$ inconsistent)

(o) Math 3A was 3A-wesome!!!

(Telugu: 15 = Padihenu)

TRUE!!!!

And with this, I'd like to thank you for flying Peyam Airlines, I hope you had a pleasant stay on board, and I wish you a safe onward journey!!!