ID: Name:

Quiz 1

No calculator, textbook or note allowed. Write your name and ID number on the front of the quiz. **Show all your work for full credit.**

1. (10 Points) Verify by direct substitution that

$$u(x,y) = -\frac{1}{2\pi}\log(\sqrt{x^2+y^2})$$

is a solution of $u_{xx} + u_{yy} = 0$. It is the fundamental solution of Laplace's equation.

Sol: $U_x = -\frac{1}{2\pi}\cdot\left(\frac{1}{\sqrt{x^2+y^2}}\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{x^2+y^2}}\cdot 2x\right)$

$= -\frac{1}{2\pi}\cdot\frac{x}{x^2+y^2}.$

$U_{xx} = -\frac{1}{2\pi}\cdot\left(\frac{1}{x^2+y^2} - \frac{2x\cdot x}{(x^2+y^2)^2}\right)$

$= -\frac{1}{2\pi}\cdot\left(\frac{x^2+y^2}{(x^2+y^2)^2} - \frac{2x^2}{(x^2+y^2)}\right)$

$= -\frac{1}{2\pi}\cdot\frac{y^2-x^2}{(x^2+y^2)^2}$

Since $U(x,y)$ is symmetric w/ respect to x & y,

$U_{yy} = -\frac{1}{2\pi}\cdot\frac{x^2-y^2}{(x^2+y^2)^2}$

So $U_{xx} + U_{yy} = -\frac{1}{2\pi}\cdot\left(\frac{y^2-x^2}{(x^2+y^2)^2} + \frac{x^2-y^2}{(x^2+y^2)^2}\right)$

$= 0.$