Quiz 4

No calculator, textbook or note allowed. Write your name and ID number on the front of the quiz. **Show all your work for full credit.**

1. (10 Points) Consider the following Dirichlet problem for diffusion equation,

$$\begin{cases} u_t = u_{xx}, & \text{in } \{0 \le x \le 2, 0 \le t \le \infty\} \\ u(0,t) = u(2,t) = 0, \\ u(x,0) = (2-x)x. \end{cases}$$

Prove that, if u(x,t) solves the Dirichlet problem, then $\tilde{u}(x,t) = u(2-x,t)$ solves it as well.

Pf:

$$\tilde{u}_x = -u_x, \qquad \tilde{u}_t = u_t.$$

$$\tilde{u}_x = u_{xx},$$

$$\tilde{u}(0,t) = u(2,t) = 0,$$

$$\tilde{u}(2,t) = u(0,t) = 0,$$

$$\tilde{u}(x,0) = (2-(2-x))\cdot(2-x)$$

$$= (2-x)\cdot x$$

Hence, $\tilde{u}$ solves $\begin{cases} u_t = u_{xx} \\ u(0,t) = u(2,t) = 0 \\ u(x,0) = (2-x)x. \end{cases}$