No calculator, textbook or note allowed. Write your name and ID number on the front of the quiz. **Show all your work for full credit.**

1. (10 Points) Consider the following Dirichlet problem for diffusion equation,

\[
\begin{cases}
    u_t = u_{xx}, & \text{in } \{0 \leq x \leq 2, 0 \leq t \leq \infty\} \\
    u(0,t) = u(2,t) = 0, \\
    u(x,0) = (2-x)x.
\end{cases}
\]

Prove that, if \( u(x,t) \) solves the Dirichlet problem, then \( \tilde{u}(x,t) = u(2-x,t) \) solves it as well.

**Pf:**

\[
\begin{align*}
    \tilde{u}_x &= -u_x, \quad \tilde{u}_t = u_t. \\
    \tilde{u}_x &= u_{xx}, \\
    \tilde{u}(0,t) &= u(2,t) = 0, \\
    \tilde{u}(2,t) &= u(0,t) = 0, \\
    \tilde{u}(x,0) &= (2-(2-x))(2-x) \\
    &= (2-x)x.
\end{align*}
\]

Hence, \( \tilde{u} \) solves

\[
\begin{cases}
    u_t = u_{xx} \\
    u(0,t) = u(2,t) = 0 \\
    u(x,0) = (2-x)x.
\end{cases}
\]