ID: Name:

Quiz 5

No calculator, textbook or note allowed. Write your name and ID number on the front of the quiz. **Show all your work for full credit.**

1. (10 Points) Derive the formula for the following Neumann problem for $x > 0$,

$$\begin{cases} u_t = ku_{xx} \ , \\ u_x(0,t) = 2 \ , \\ u(x,0) = \varphi(x). \end{cases}$$

(HINT: Do **NOT** simplify the resulting expression, but please write it in terms of φ.)

Sol: Let $v(x,t) = u(x,t) - 2x$,

then v solves $\begin{cases} v_t = kv_{xx}, \ 0 < x < \infty. \\ v_x(0,t) = 0, \\ v(x,0) = \varphi(x) - 2x. \end{cases}$

Need to use even extension for Neumann Problem, so $\varphi_{even} = \begin{cases} \varphi(x) - 2x, & x \geq 0 \\ \varphi(-x) + 2x, & x < 0 \end{cases}$

So, $v(x,t) = \int_0^\infty (S(x-y,t) + S(x+y,t)) \varphi_{even}(y) dy$

$= \int_0^\infty (S(x-y,t) + S(x+y,t))(\varphi(y) - 2y) dy$

So $u(x,t) = \int_0^\infty (S(x-y,t) + S(x+y,t))(\varphi(y) - 2y) dy$

$+ 2x.$

[redacted]