Quiz 5

No calculator, textbook or note allowed. Write your name and ID number on the front of the quiz. Show all your work for full credit.

1. (10 Points) Derive the formula for the following Neumann problem for $x > 0$,

$$
\begin{cases}
  u_t = ku_{xx}, \\
  u_x(0,t) = 2, \\
  u(x,0) = \varphi(x).
\end{cases}
$$

(HINT: Do NOT simplify the resulting expression, but please write it in terms of $\varphi$.)

Sol: Let $v(x,t) = u(x,t) - 2x$,

then $v$ solves

$$
\begin{cases}
  v_t = kv_{xx}, & 0 < x < \infty, \\
  v_x(0,t) = 0, \\
  v(x,0) = \varphi(x) - 2x.
\end{cases}
$$

Need to use even extension for Neumann Problem, so $\Phi_{even} = \begin{cases}
  \Phi(x) - 2x, & x \geq 0 \\
  \Phi(-x) + 2x, & x < 0
\end{cases}$

So, $v(x,t) = \int_0^\infty (S(x-y,t) + S(x+y,t)) \Phi_{even}(y) \, dy$

$$
= \int_0^\infty (S(x-y,t) + S(x+y,t)) (\varphi(y) - 2y) \, dy
$$

So $u(x,t) = \int_0^\infty (S(x-y,t) + S(x+y,t)) (\varphi(y) - 2y) \, dy + 2x$.  

(Math 112A)