

Quiz 6

No calculator, textbook or note allowed. Write your name and ID number on the front of the quiz. **Show all your work for full credit.**

1. (10 Points) Find the solution of,

$$\begin{cases} u_t = ku_{xx}, & 0 < x < \pi \\ u(0, t) = u(\pi, t) = 0, \\ u(x, 0) = 3 \sin 2x + 4 \sin 3x. \end{cases}$$

(HINT: Just assume λ is less than 0, no need to discuss all cases.)

Sol: Let $u(x, t) = X(x)T(t)$

$$\text{Then } X \cdot T' = kX''T, \Rightarrow \frac{T'}{kT} = \frac{X''}{X} = \lambda < 0.$$

$$\text{Hence } X(x) = A \cos(\sqrt{\lambda}x) + B \sin(\sqrt{\lambda}x).$$

$$X(0) = A = 0, \quad X(\pi) = B \sin(\sqrt{\lambda}\pi) = 0$$

$$\Rightarrow \sqrt{\lambda} = n, \quad n = 1, 2, \dots, \quad \text{so} \quad -\lambda = n^2,$$

$$\text{Then } \frac{T'}{kT} = -n^2, \Rightarrow T = C e^{-kn^2 t}, \quad C \text{ is a const.}$$

$$\text{So } u(x, t) = \sum_{n=1}^{\infty} B_n \sin(nx) \cdot e^{-kn^2 t}.$$

$$\text{and } u(x, 0) = \sum_{n=1}^{\infty} B_n \sin(nx) = 3 \sin 2x + 4 \sin 3x$$

$$\Rightarrow B_2 = 3, \quad B_3 = 4, \quad B_n = 0, \quad \text{for } n \neq 2, 3.$$

$$\text{So } u(x, t) = 3 \sin(2x) e^{-4kt} + 4 \sin(3x) e^{-9kt}.$$

