

ID:

Name:

## Quiz 8

No calculator, textbook or note allowed. Write your name and ID number on the front of the quiz. **Show all your work for full credit.**

1. (10 Points) Find the solution of

$$\begin{cases} \Delta u = 0, \text{ in } [0, \pi] \times [0, \pi] \\ u_y(x, 0) = 0, u_y(x, \pi) = 0 \\ u(0, y) = 0, u(\pi, y) = \frac{1}{2}(1 + \cos 2y). \end{cases}$$

Sol:  $\begin{cases} Y'' = -\lambda Y \\ Y'(0) = Y'(\pi) = 0 \end{cases} \Rightarrow$  Case 1:  $\lambda = 0$   
 $Y = B \neq 0.$   
Case 2:  $\lambda > 0.$   
 $\Rightarrow \lambda = n^2$   
 $Y = A_n \cos(ny), n=1, 2, \dots$

Then  $\begin{cases} X'' = n^2 X \\ X(0) = 0 \end{cases} \Rightarrow \begin{cases} X = B_n e^{nx} - B_n e^{-nx}, n=1, 2, \dots \\ X = A_n x, n=0. \end{cases}$

So  $u = X \cdot Y = A_0 x + \sum_{n=1}^{\infty} A_n \cos(ny)(e^{nx} - e^{-nx})$

$$u(\pi, y) = A_0 \pi + \sum_{n=1}^{\infty} A_n \cos(ny)(e^{n\pi} - e^{-n\pi})$$

$$= \frac{1}{2} + \frac{1}{2} \cos 2y$$

$$\Rightarrow A_0 = \frac{1}{2\pi}, A_2 = \frac{1}{2} \cdot \frac{1}{e^{2\pi} - e^{-2\pi}}, A_n = 0, n \neq 0, 2.$$

So  $u(x, y) = \frac{1}{2\pi} x + \frac{1}{2} \cdot \frac{1}{e^{2\pi} - e^{-2\pi}} \cdot (e^{nx} - e^{-nx}) \cdot \cos 2y$