LECTURE 1 - SYSTEMS OF EQUATIONS

Sunday, September 15, 2019 6:00 PM

I- INTRODUCTION

Hello everyone and welcome to Math 3A! My name is Peyam and I'll be your instructor this quarter!
First of all, notice the Pi in my name, and that's because I love Math and I love food.

By the way, I know people like to call me Prof. Tabrizian, but please, call me Peyam… or Dr. Peyam, like my awesome YouTube channel, which hit 30,000 subscribers last week!

Logistics: All the info is on the syllabus, which can be found on my website

Course: This course meets MWF here, but whether you show up is up to you. There's also discussion section, which is on Tu/Th, and which is optional

OH: W 12:30 - 1:30 pm, Th 2-3 pm. Please come, I'd be happy to help!

Textbook: When I TA-ed for 3A at Berkeley, it was called Lay, but now it's called Lay-Lay-McDonald. It's pretty good, read it!

Grading:

- HW 0 %
- Quizzes 20 %, every Th during discussion section, **including** the week of the midterm. Lowest 2 quizzes dropped. No quiz during Week 0 or during Thanksgiving week
- Midterm 30%, on Friday November 1 in class. If your final exam score is better than your midterm score, then it can replace your midterm score
- Final 50 %, on Friday, December 13, 1:30 - 3:30 pm, cumulative

Grades: This class is (probably) going to be curved. I will assign grades according to the standard math department curve, which is: 20 % A, 25 % B, 30 % C, 15 % D, 10 % F

II- SYSTEMS OF LINEAR EQUATIONS

WHAT IS LINEAR ALGEBRA? It's the math of solving systems of (linear) equations. And in fact, let me lay all my cards on the table and tell you the most important technique in this course (= Master Sword of Linear Algebra)

Example: Solve the following system

$$\begin{cases} x + 2y + z = 0 \\ x - y + 3z = -2 \\ 3x + 0y - z = 4 \end{cases}$$

Notice: The names of the variables doesn't matter at all! You could write the system as follows, and still have the same system:

$$\begin{cases} \text{☺} + 2\heartsuit + \text{D} = 0 \\ \text{☺} - \heartsuit + 3\text{D} = -2 \\ 3\text{☺} + 0\heartsuit - \text{D} = 4 \end{cases}$$

SAME SYSTEM!

In fact... let's forget about the variables altogether!

STEP 1: Write the system in "Matrix form" (all this means is: Put the coefficients in a table)

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 1 & -1 & 3 & -2 \\ 3 & 0 & -1 & 4 \end{array}\right] \leftarrow \text{"AUGMENTED" MATRIX}$$

(what this means is that we don't care about the variables, just about the numbers in front of them)

STEP 2: Transform the matrix into *Triangular* form

ALLOWABLE MOVES: (Eros = Elementary Row Operations)

1) Interchange 2 rows $\left[\begin{array}{cc} 1 & 2 \\ 3 & 4 \end{array}\right] \rightarrow \left[\begin{array}{cc} 3 & 4 \\ 1 & 2 \end{array}\right]$

2) Multiply/Divide a row by a constant $(\times 2)\ \left[\begin{array}{cc} 1 & 2 \\ 3 & 4 \end{array}\right] \rightarrow \left[\begin{array}{cc} 2 & 4 \\ 3 & 4 \end{array}\right]$

3) Add/Subtract a constant times a row from another $(\times -3)\ \left[\begin{array}{cc} 1 & 2 \\ 3 & 4 \end{array}\right] \rightarrow \left[\begin{array}{cc} 1 & 2 \\ 0 & -2 \end{array}\right]$

OK, so now let's do it, let's row-reduce this matrix (like tetris)

$$(\times -1)\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 1 & -1 & 3 & -2 \\ 3 & 0 & -1 & 4 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -3 & 2 & -2 \\ 3 & 0 & -1 & 4 \end{array}\right](\times -3)$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -3 & 2 & -2 \\ 0 & -6 & 4 & 4 \end{array}\right] \quad (x-2)$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -3 & 2 & -2 \\ 0 & 0 & -8 & 8 \end{array}\right]$$ TRIANGULAR FORM

Note: 1,-3,-8 are called pivots (see next lecture)

STEP 3: BACKSUBSTITUTION (what does that mean in terms of x, y, z)

$$\begin{cases} x + 2y + z = 0 \\ -3y + 2z = -2 \\ -8z = 8 \end{cases} \rightsquigarrow$$

$z = -1$

$-3y = -2 - 2z = -2 - 2(-1) = 0 \Rightarrow y = 0$

$x = -2y - z = -2(0) - (-1) = 1 \Rightarrow x = 1$

$\Rightarrow$ SOLUTION $\boxed{x = 1,\ y = 0,\ z = -1}$ TA-DAAA!!!

POINT: Using this technique (called row-reduction), we transformed the system into one that's much easier to solve!

CANNOT emphasize how important this is!

ALL we're going to do this quarter is row-reduce!

And in fact, if you're ever stuck on a linear algebra question, just row reduce and it'll give you the answer (or at least partial credit)

III- INCONSISTENT SYSTEMS

Ok, now that we've seen how wonderful this is, let's solve the following system

Example: "Solve"

$$\begin{cases} y + 4z = -5 \\ x + 3y + 5z = -2 \\ 3x + 7y + 7z = 6 \end{cases}$$

1) $$\left[\begin{array}{ccc|c} 0 & 1 & 4 & -5 \\ 1 & 3 & 5 & -2 \\ 3 & 7 & 7 & 6 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 3 & 7 & 7 & 6 \\ 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -5 \end{array}\right]$$ (better to have 0's at the bottom)

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 5 & -2 \\ 3 & 7 & 7 & 6 \\ 0 & 1 & 4 & -5 \end{array}\right] (x-3)$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 5 & -2 \\ 0 & -2 & -8 & 12 \\ 0 & 1 & 4 & -5 \end{array}\right] (\div -2)$$

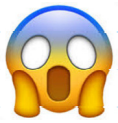
$$\rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -6 \\ 0 & 1 & 4 & -5 \end{array}\right] (x-1)$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -6 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

(Still in triangular form,
But what happened here???)

3) Backsubstitution

$$\begin{cases} x + 3y + 5z = -2 \\ y + 4z = -6 \\ 0 = 1 \end{cases}$$



CONTRADICTION !!!!!!!

In other words, there is no way this system has a solution, so it's INCONSISTENT

You might ask: Could there be other ways a system might have no solution?
Fortunately the answer is NO

IMPORTANT FACT: The ONLY way a system is inconsistent is if one row of the augmented matrix looks like this:

$$[0\ 0\ 0\ \cdots\ 0 \mid \text{BLAH}] \quad \text{WITH} \quad \text{BLAH} \neq 0$$

Here: $[0\ 0\ 0 \mid 1]$ AND $1 \neq 0 \Rightarrow$ INCONSISTENT

Example: Find all h for which the following system is inconsistent

$$\begin{cases} x + hy = -5 \\ 2x - 8y = 6 \end{cases}$$

$$(\div 2)\begin{bmatrix} 1 & h & -5 \\ 2 & -8 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & h & -5 \\ 1 & -4 & 3 \end{bmatrix} \downarrow (\times -1)$$

$$\rightarrow \begin{bmatrix} 1 & h & -5 \\ 0 & -h-4 & 8 \end{bmatrix} \qquad [0\ 0\ |\ 8]$$

By FACT, inconsistent if and only if $-h - 4 = 0 \Rightarrow \boxed{h = -4}$