LECTURE 1: DOUBLE INTEGRALS

1. Introduction

- Hello everyone and welcome to Math 2E! My name is Peyam and I'll be your instructor this quarter!

- First of all, notice the π in my name, and that's because I love Math and I love food.

- By the way, I know people like to call me Prof. Tabrizian, but please, call me Peyam... or Dr. Peyam, like my awesome YouTube channel, which has almost 38,000 subscribers!

- **Logistics:** All the info is on the syllabus, which can be found on my website

- **Course:** This course meets MWF here, but whether you show up is up to you. You can show up to any lecture that you want, it doesn't matter. There's also discussion section, which is on Tu/Th, and which is optional

Date: Monday, January 6, 2020.

- **OH:** W 12 - 1:30 pm in 410N Rowland. Please come, I'd be happy to help! Also, **VIRTUAL OH** W 6:30 - 7 pm via YouTube (Very easy: I send you a link, you click it, and you're virtually teleported to my office)

- **Textbook:** Stewart Calculus; complete garbage, don't read it

- **Grading:**
  - HW 0 %
  - Quizzes 20 % every week, every Th during discussion section, lowest 2 quizzes dropped (Q: Do we have a quiz this week? A: Yes)
  - Midterm 30 %, on Friday, February 7 in class. If your final exam score is better than your midterm score, then it can replace your midterm score
  - Final 50 %, cumulative on Monday, March 16, 10:30 - 12:30 pm (for LEC F) or Friday, March 20, 8 - 10 am (for LEC A)

- **Grades:** This class is going to be curved. I will assign grades according to the standard math department curve, which is: 20 % A, 25 % B, 30 % C, 15 % D, 10 % F. There is no such thing as an easy professor, we're all supposed to follow this curve.

- **Finally:** If you don't like this course, I understand. When I took it at Cal in freshman year, I hated this course (and **gasp**

I got a B+ in it). But when I taught this course in Fall 2018, I completely fell in love with it, and my goal this quarter is to share this love with you :)

2. Double Integrals

Previously in Math 2D: Learned about this AMAZING tool called Double Integrals, which allows us to calculate volumes with a snap of a finger. And in fact, this course is all about integration: We'll do double integrals, triple integrals, we'll even integrate over curves and surfaces (WOW).

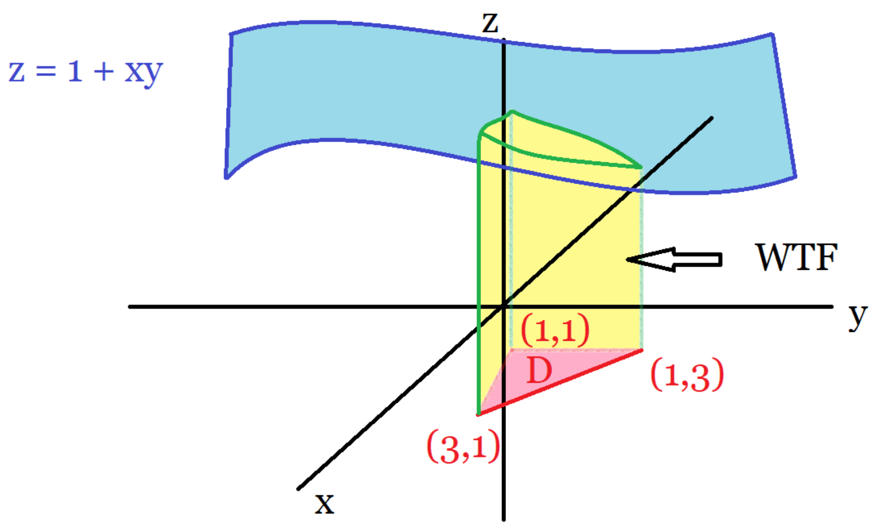
Today: Let's start chill and do some double integrals

Example: Find

$$\int\int_D 1 + xy \, dxdy$$

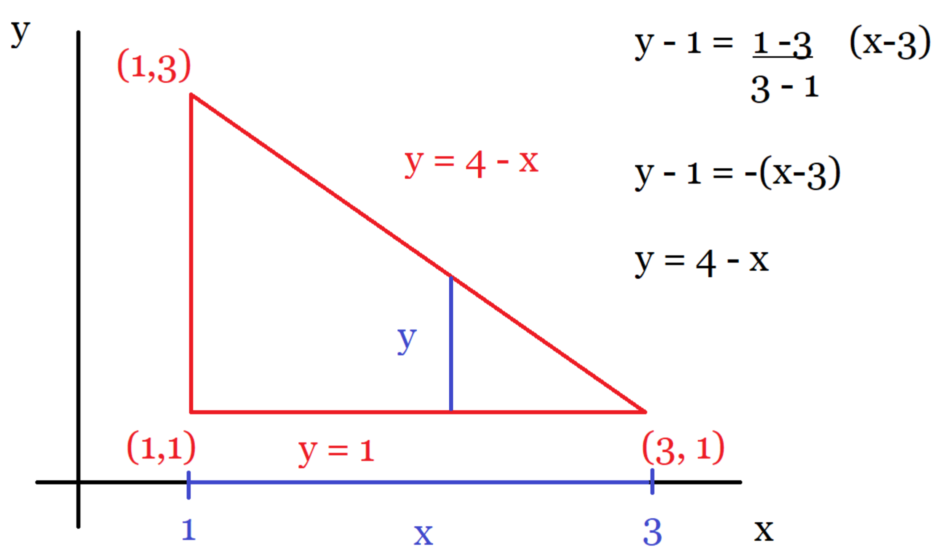
where D is the triangle with vertices $(1,1),(1,3),(3,1)$

(1) **Picture**



Note: WTF = Want To Find

(2) **Draw D**



(3)

$$\text{Smaller } \leq y \leq \text{ Bigger } \Rightarrow 1 \leq y \leq 4 - x$$

$$\text{Left } \leq x \leq \text{ Right } \Rightarrow 1 \leq x \leq 3$$

(4)

$$\begin{aligned}\int\int_D 1 + xy\, dxdy =& \int_1^3 \int_1^{4-x} 1 + xy \; dydx \; \text{ Beware of the order} \\ =& \int_1^3 \left[y + \frac{1}{2} xy^2 \right]_{y=1}^{y=4-x} dx \\ =& \int_1^3 4 - x + \frac{x}{2}(4-x)^2 - 1 - \frac{x}{2} dx \\ =& \cdots \\ =& \frac{22}{3}\end{aligned}$$

Interpretation: The volume under $z = 1 + xy$ and over D is $\frac{22}{3}$.

3. Polar Coordinates

Video: Volume of an Ice Cream Cone

Example: Find the volume of the region between $z = \sqrt{x^2 + y^2}$ and $z = \sqrt{8 - (x^2 + y^2)}$

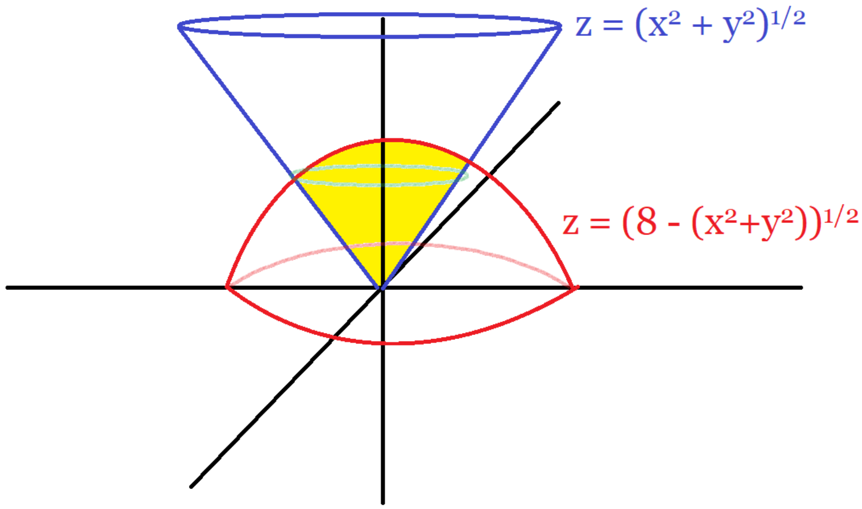
(1) **Picture:**

Note: $z = \sqrt{x^2+y^2} \Rightarrow z^2 = x^2 + y^2$ **CONE**.

WARNING: KNOW YOUR SURFACES FROM 12.6 !!!

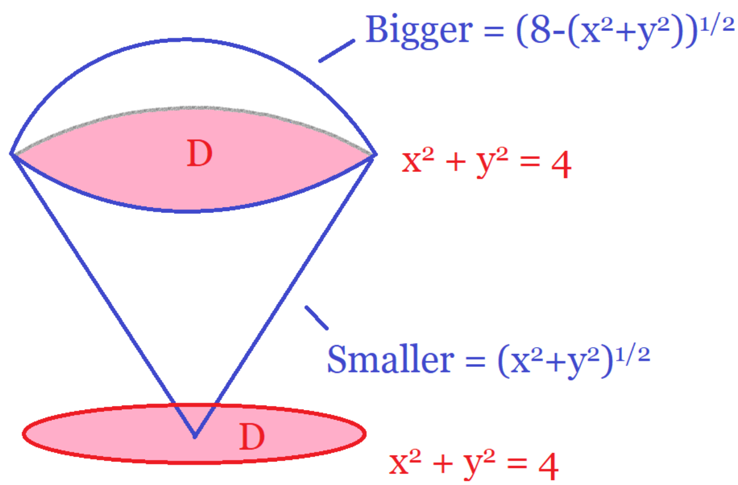
(But since $z \geq 0$, actually upper-half cone)

$z = \sqrt{8-(x^2+y^2)} \Rightarrow z^2 = 8-(x^2+y^2) \Rightarrow x^2+y^2+z^2 = 8$ SPHERE



This looks messy, so let's redraw it

(2) **Re-draw:**



(3)

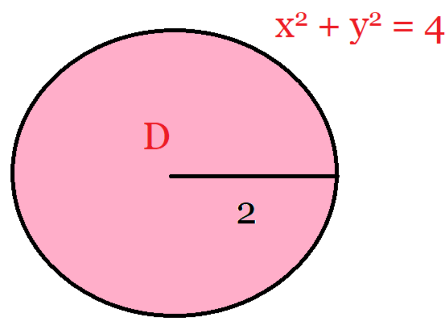
$$V = \int\int_D \text{Bigger} - \text{Smaller } dxdy = \int\int_D \sqrt{8 - x^2 - y^2} - \sqrt{x^2 + y^2} dxdy$$

(4) **Find D**

Intersection:

$$\begin{aligned} \sqrt{8 - x^2 - y^2} &= \sqrt{x^2 + y^2} \\ 8 - x^2 - y^2 &= x^2 + y^2 \\ 8 &= 2(x^2 + y^2) \\ x^2 + y^2 &= 4 \end{aligned}$$

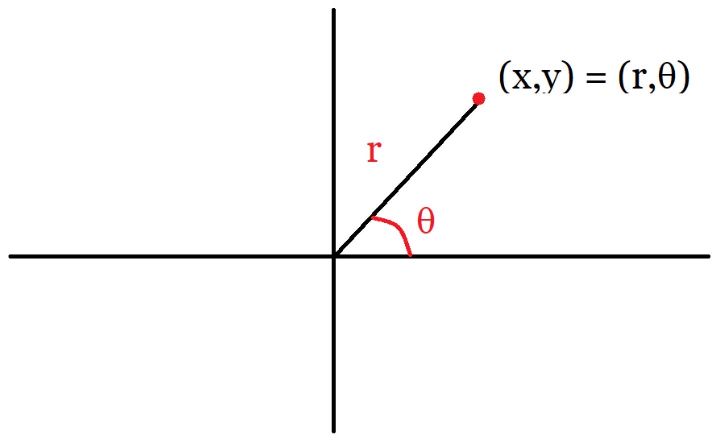
So $D =$ Disk of radius 2



(5) **Polar Coordinates**

Rule of Thumb: If D is circular and/or you see $x^2 + y^2$, use polar coordinates.

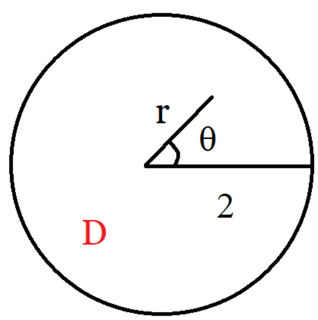
Polar Coordinates:



(a)

$$\sqrt{8-(x^2+y^2)}-\sqrt{x^2+y^2}=\sqrt{8-r^2}-r$$

(b) D: $0 \leq r \leq 2, 0 \leq \theta \leq 2\pi$



(6)

$$
\begin{aligned}
V =& \int_0^{2\pi} \int_0^2 \left[\sqrt{8-r^2} - r\right] r \, dr d\theta \quad \text{DON'T FORGET ABOUT r !!!!!!!!!!!!} \\
=& 2\pi \int_0^2 \left(\sqrt{8-r^2}\right) r - r^2 dr \\
& (u = 8 - r^2, du = -2rdr \Rightarrow rdr = -\frac{1}{2}du, u(0) = 8, u(2) = 4) \\
=& 2\pi \left(\int_8^4 \sqrt{u} \left(-\frac{1}{2}du\right) - \left[\frac{1}{3}r^3\right]_0^2 \right) \\
=& 2\pi \left(\left[\frac{2}{3}u^{\frac{3}{2}}\right]_4^8 - \frac{8}{3} \right) \\
=& \cdots \\
=& \frac{32}{3}\pi \left(\sqrt{2} - 1\right)
\end{aligned}
$$