MIDTERM – STUDY GUIDE

The Midterm takes place on **Friday, February 7, 2020** during your usual lecture time and in your usual lecture room. **You have to take the exam in your lecture that you're officially enrolled in, not the one you usually attend.** The two lectures will be curved separately, so it doesn't matter if one exam is easier than the other one. **NO** books/notes/calculators/cheat sheets will be allowed. **Please bring your ID, for verification purposes.** It counts for 30 $\%$ of your grade, and covers sections 15.2 – 15.9, and sections 16.1 – 16.3 *inclusive*. It will **NOT** cover section 16.4 (Green's theorem). This is the study guide for the exam, and is just meant to be a *guide* to help you study, just so we're on the same place in terms of expectations. For a more thorough study experience, look at all the lectures, the practice exam, the actual midterm from 2018, and the suggested homework.

The exam will *tentatively* have 5-6 problems in total, so roughly 10 minutes per question.

IMPORTANT: You **ABSOLUTELY** need to know the 6 surfaces on Table 1 of section 12.6; I could ask you about any of those surfaces! And don't think you can get away by now drawing a picture, I could force you to draw one!

On the exam, I *will* give you the following equations for spherical coordinates, so you do **NOT** have to memorize them:

$$\begin{aligned} x &= \rho \sin(\phi)\cos(\theta) \\ y &= \rho \sin(\phi)\sin(\theta) \\ z &= \rho \cos(\phi) \\ \text{Jac} &= \rho^2 \sin(\phi) \end{aligned}$$

Useful trig identities to know:

(1) $\sin^2(x) + \cos^2(x) = 1$
(2) $1 + \tan^2(x) = \sec^2(x)$
(3) $\cos(-x) = \cos(x), \sin(-x) = -\sin(x)$
(4) $\sin(2x) = 2\sin(x)\cos(x)$, $\cos(2x) = \cos^2(x) - \sin^2(x)$

Useful integration techniques to know:

(1) $u-$substitution

(2) Integration by parts, but I'll only ask you about easy cases like $\int xe^x$
(3) Trig integrals like $\int \cos^3(x)\sin(x)dx$
(4) $\int \cos^2(x)dx$, $\int \sin^2(x)dx$
(5) **No** trig substitutions or partial fractions needed for the exam

Section 15.2: Double Integrals over General Regions

- Find the double integral of a function over a general region. Sometimes the region is a vertical region, so you'll have to do Smaller $\leq y \leq$ Bigger, and sometimes it's a horizontal region, in which case you have to solve for x in terms of y and do Left $\leq x \leq$ Right.
- Find the volume of a given solid (like 15.2.25 or 15.2.27)
- Change the order of integration; for example, write $\int_0^1 \int_x^1 \sin(y^2)dydx$ as $dxdy$ and evaluate the integral, see this video.
- Find the average value of a function (15.2.62)

Section 15.3: Double Integrals in Polar Coordinates

- You only need to know Formula 2 on top of page 1012, as well as Examples 1 and 2; ignore Examples 3 and 4. The hardest trig integral I can ask you is $\int \sin^2(x)$ or $\int \cos^2(x)$, I won't ask you about $\int \sin^4(x)$ or stuff like that.
- Evaluate an integral by changing to polar coordinates. Remember that this is excellent if you see $x^2 + y^2$ or your region is a disk or a wedge or a ring (annulus). **Don't forget about the r!!!**, check out this video and this video
- Find the volume between two surfaces; in particular remember the ice cream cone problem I did in lecture, it's a good practice for 15.6 (check out this YouTube video)
- Calculate $\int_{-\infty}^{\infty} e^{-x^2}dx$ (you can check out this YouTube video). Don't worry about the problem with $\int_{-\infty}^{\infty} \sin(x^2)dx$

Section 15.6: Triple Integrals

- You **don't** need to know the definition of the integral or Fubini's theorem (but of course know how to use it) and you don't need to know the names of the regions (and personally I find the formulas on page 1031 confusing). Finally, the only applications you need to know are Formula 12 on page 1034 (volume) and the average value (exercise 53), you are **not** responsible for moments, center of mass, etc. If I ask you about mass, I would give you the formula (just like on the practice exam)

- Find a triple integral over a general $3D$ region; good examples of regions are either tetrahedrons, or regions between two surfaces (like in lecture); *usually* you have to do Smaller $\leq z \leq$ Bigger
- Find the average value of a function over a $3D$ region (like 15.6.54)
- Remember that sometimes you region faces the $y-$ direction (the book calls this type 3), in which case you have to do Left $\leq y \leq$ Right or the $x-$direction (type 2), in which case you have to do Back $\leq x \leq$ Front.
- Sketch the solid whose volume is given by a given integral.
- You do **NOT** need to know how to change the order of integration for a triple integral
- Review the problems from Lecture 3, those are very good sample questions
- In particular, know how to find the volume of intersection of 2 cylinders, like in this video. Note: The solution I presented in lecture was wrong, but I wrote up the correct solution in the notes. You are still responsible for knowing it.

Section 15.7: Triple Integrals in Cylindrical Coordinates

- There is absolutely nothing new to learn in this section, it's basically polar coordinates, except you add a z.
- Calculate triple integrals using cylindrical coordinates; problems 17 through 24 are excellent practice problems
- Remember that cylindrical coordinates are useful if you have cylinders or if you see $\sqrt{x^2+y^2}$
- Gotta graph them all, gotta graph them all, Surfaces!!!

Section 15.8: Triple Integrals in Spherical Coordinates

- Know how to rederive the equations for spherical coordinates, see this video.
- Again, I will provide you with the equations for spherical coordinates, so no need to memorize them
- Plot a point with given spherical coordinates
- Sketch surfaces with given spherical coordinates, like $\rho = 3$ or $\phi = \frac{\pi}{6}$
- Calculate integrals and volumes using spherical coordinates; problems 21, $22-28$, and 30 are excellent practice problems. The problems in lecture are also great, and you can check out this YouTube video.
- Remember that spherical coordinates are great for spheres and anything that involves $\sqrt{x^2+y^2+z^2}$.

- I could ask you to calculate the mass of the sun, although of course I will give you the density, see this video.

SECTION 15.9: CHANGE OF VARIABLES IN MULTIPLE INTEGRALS

- There are two kinds of change of variables I could ask you:
- One where you choose u and v, like in $\int\int \cos\left(\frac{y-x}{y+x}\right) dxdy$. In that case, I will **NOT** give you u and v and you'll have to figure it out, either by using the function, or by using the region (like in the problem on the practice exam). Problems $23 - 28$ are great sample problems, as well as the problems in lecture and on the 2018 midterm.
- Or one where x and y are in terms of u and v, like Problems $15 - 19$. In that case, I will **give** you x and y.
- Don't forget about the absolute value!!!
- I could ask you about double or triple integrals
- The easiest way to memorize the Jacobian definition is by using $dx = \frac{dx}{du}du$, or $du = \frac{du}{dx}dx$. Remember the motivating examples in lecture!
- Know how to rederive the Jacobians for polar, cylindrical, and spherical coordinates (see Lecture 8 or the Additional Problems in HW 3)
- Know how to find the area of an ellipse and the volume of an ellipsoid, just like on the Additional Problems in HW 3.
- You might also want to check out my YouTube videos: Part 1, Part 2, and Part 3.
- The last problem on the 2018 midterm is fair game on the exam, although I would of course give you all the identities for hyperbolic functions. See this video.

SECTION 16.1: VECTOR FIELDS

- I won't ask you anything about this section since it's just an introduction, so you can skip it if you want
- Here's a good overview of Chapter 16: Vector Calculus Overview

SECTION 16.2: LINE INTEGRALS

- Remember the three important parametrizations in Lecture 10: The circle, the line segment, and the function, see this video.
- Calculate $\int_C f(x,y)ds$ or $\int_C f(x,y,z)ds$, like problems 3, 4, $9 - 12$. And remember that it represents the area under a fence, see this video, this video, and this video for examples.

- The easiest way to memorize this is by using $ds = \sqrt{(dx)^2 + (dy)^2}$ (in the two-dimensional case).
- I could even ask you to rederive the formula for line integrals, see this video.
- Calculate $\int_C Pdx + Qdy$ or $\int_C Pdx + Qdy + Rdz$, like problems 5, 6, 8, 13 -16, see this video
- The easiest way to memorize this is by using $dx = \frac{dx}{dt}dt = x'(t)dt$
- You don't need to know the interpretation given in Lecture 11 with the shadows
- Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, like problems 19-22, see this video.
- Find the work done by a vector field $\mathbf{F}$ on a curve C, like $40 - 41$

SECTION 16.3: THE FUNDAMENTAL THEOREM FOR LINE INTEGRALS

- Determine if a vector field $\mathbf{F}$ is conservative, like in $3 - 10$. This question only works in the $2D$ case.
- Show that a vector field is not conservative given a picture, like in 25. This just amounts in finding a curve C where $\int_C \mathbf{F} \cdot d\mathbf{r} \neq 0$
- Find a function f such that $\mathbf{F} = \nabla f$, like in $12 - 18$. This works in any dimensions. You can use the method in lecture or in the book, whichever you prefer.
- Know that in $2D$, $\mathbf{F} = \langle P, Q \rangle$ is conservative if and only if $P_y = Q_x$. Remember the mnemonic Peyam is Quixotic. Don't worry about simply-connectedness, I won't trick you with that.
- Use the FTC for line integrals to calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, like in $12 - 18$, see this video
- Know that $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of path if and only if $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ for any closed curve C if and only if $\mathbf{F}$ is conservative. See also problems 19 and 20.
- Ignore problems 29 and 35.
- The problems in Lecture 13 and on the 2018 midterm are good problems as well.