Math 2E – Suggested Homework 7

Peyam Tabrizian

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Reading: Section 16.6. We will do section 16.5 later, when we'll talk about Stokes' Theorem and the Divergence Theorem. In 16.6, you can ignore all the sections with computer algebra systems.

Note: No lecture on Monday, February 17 because of Presidents' Day and my dad's 91st birthday ☺

- **Section 16.6:** 20, 23, 24, 26, 33, 35, 40, 41, 45, 47, 48, 59(a)(c), AP1, AP2, (Optional: 64(a)(c))

Additional Problem 1:

(a) Suppose S is the surface obtained by rotating the graph of $y = f(x)$ from $x = a$ to $x = b$ about the $x-$axis. Find a formula for the surface area of S.

Hint: Use the parametric equations $x = x$, $y = f(x)\cos(\theta)$, $z = f(x)\sin(\theta)$, $a \leq x \leq b$, $0 \leq \theta \leq 2\pi$ (see Example 5 in Lecture 17)

(b) Use your formula in (a) and the fact that $1 + \frac{1}{x^4} \geq 1$ to find the surface area obtained by rotating the graph of $y = \frac{1}{x}$ from $x = 1$ to $x = \infty$ about the $x-$axis (this is called Gabriel's horn). Optional: use the disk method (from Math 2B) to find the volume of the resulting solid. Isn't this result surprising?

Awesome consequence: You can fill Gabriel's horn with paint, but you can never paint it!

(TURN PAGE for AP2)

Additional Problem 2:

In this problem, you might need the following facts about hyperbolic trig functions (which will be provided to you on the quiz/exam):

$$\begin{aligned}
\cosh(\alpha) &= \frac{e^{\alpha} + e^{-\alpha}}{2} \\
\sinh(\alpha) &= \frac{e^{\alpha} - e^{-\alpha}}{2} \\
\cosh^2(\alpha) - \sinh^2(\alpha) &= 1 \\
(\cosh(\alpha))' &= \sinh(\alpha) \\
(\sinh(\alpha))' &= \cosh(\alpha)
\end{aligned}$$

(a) Let S be the portion of the hyperboloid of one sheet (dress) $x^2+y^2-z^2=1$ between $z=-1$ and $z=1$. Find parametric equations for S.

Hint: Start with $z=\sinh(\alpha)$ and find x and y such that $x^2+y^2=\cosh^2(\alpha)$. Think polar coordinates.

(b) Set up, but do **NOT** evaluate, an integral for the surface area for S.