MIDTERM REVIEW PROBLEMS

(1) Use the following change of variables to calculate

$$\int\int_D x^2 - xy + y^2 dxdy$$

Where D is the region enclosed by the ellipse $x^2 - xy + y^2 = 2$

$$\begin{cases} x = \sqrt{2}u - \sqrt{\dfrac{2}{3}}v \\ y = \sqrt{2}u + \sqrt{\dfrac{2}{3}}v \end{cases}$$

Solution: Change of Variables

(2) Evaluate

$$\int\int\int_E z dxdydz$$

Where E is the region inside the sphere $x^2 + y^2 + z^2 = 1$ and above the cone $z = \sqrt{x^2 + y^2}$

Solution: Spherical Coordinates

(3) Evaluate

$$\int_C (x^2 + y^2) ds$$

Where C : line from $(1, 0)$ to $(4, 1)$

Solution: Line Integral

(4) Calculate

$$\int \int \int_E z\,dx\,dy\,dz$$

Where E is the solid in the first octant under $4x + 2y + z = 4$

Solution: Triple Integrals

(5) Calculate

$$\int \int \int_E \sqrt{x^2 + z^2}\,dx\,dy\,dz$$

Where E is the region bounded by $y = 4 - x^2 - z^2$ and the xz plane.

Solution: Cylindrical Coordinates