Math 2E – Suggested Homework 10

Peyam Tabrizian

Thursday, March 12, 2018

Reading: Sections 16.5 and 16.8. This time, in section 16.5, only focus on the part about curl. In section 16.8, you only need to know how to do Examples 1 and 2, ignore the derivation of Stokes and the explanation following formula 3.

Careful: For Stokes' theorem, always check if the orientation of C matches that of S. Think of it as follows: Whenever you walk along the curve C, you want S to be on your left (Wa**L**k Left).

Note: Congratulations, you are done with homework! I'm very proud of you of making it this far :)

- **Section 16.5:** 1(a), 7(a), 13, 17, 19, 20, 21, 26, AP1, AP2
- **Section 16.8:** 2, 3, 4, 5, 7, 9, 10, 19[1], AP3, AP4

Additional Problem 1: Show that if $\mathbf{F} = \langle P, Q, R \rangle$ is conservative, then $\operatorname{curl} \mathbf{F} = \mathbf{0}$ in two different ways:

(a) By directly showing that $\operatorname{curl}(\nabla f) = \mathbf{0}$. Why does that answer the question?

(b) By using the trick using Clairaut's theorem from lecture to conclude that if $\mathbf{F}$ is conservative, then $Q_x = P_y, R_y = Q_z, P_z = R_x$. Why does that answer the question?

[1] Hint: S is closed, so can you use another theorem?

Additional Problem 2: Show that $\operatorname{div}(\operatorname{curl}(\mathbf{F})) = 0$

Additional Problem 3: Let S denote the portion of the sphere $x^2 + y^2 + z^2 = 1$ with $-\frac{1}{2} \leq z \leq \frac{1}{2}$, oriented outwards. Calculate $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = \langle y, -2xz, ye^z \rangle$. Beware of the orientation!

Additional Problem 4: (Optional) Look at the theorems in section 16.10 (or at the FTC handout on my website), and convince yourself that every theorem says that the integral of the derivative of the function (or vector field) on a domain is equal to the value of that function on the boundary of the domain.

Important: I will be out of town on Monday, March 9, so lecture on that day is cancelled. Given that I also owe you 2 make-up lectures, here is the lecture schedule for this week. This is all tentative and subject to change.

(1) Monday, March 9: No lecture

(2) Discussion as usual

(3) Wednesday, March 11: Lecture/OH/Virtual OH as usual

(4) Thursday, March 12: Make-up Lecture during discussion + Quiz 10 (including the 8 am discussion)

(5) Friday, March 13: Lecture as usual + Make-up lecture 4-4:50 PM in 1600 DBH