MATH 107 :
Numerical DE (ODE, PDE)

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HW : submitted via Canvas (online)
Quizzes
↳ Quiz problems will be released during Thursday discussions.
↳ Your answers must be submitted until Friday 2 pm via Canvas.
↳ Quiz problems would be similar to the HW problems.

Chap 5. IVPs for ODEs

↳ Initial - Value Problems.

$y = y(t)$

$$\begin{cases} \dfrac{dy}{dt} = f(t, y), & \text{for } a \le t \le b \\ y(a) = \alpha \end{cases}$$

initial point

end point

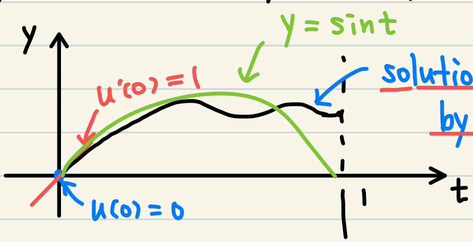
In general,

$$\begin{cases} y^{(n)} = \dfrac{d^{(n)}y}{dt^{(n)}} = f(t, y, y', \cdots, y^{(n-1)}) \\ \qquad \text{for } a \le t \le b \\ y(a) = \alpha_1, \quad y'(a) = \alpha_2, \quad \cdots, \quad y^{(n-1)}(a) = \alpha_n \end{cases}$$

$y'' = -y$

ex) $y'' + y = 0$ for $0 \le t \le 1$

$y(0) = 0$, $y'(0) = 1$



solution is determined by initial conditions.
(location, velocity)

$$y(t) = C_0 \sin t + C_1 \cos t$$
$$(\text{or } A_0 e^{it} + A_1 e^{-it})$$

$$\left.\begin{aligned} y(0) &= C_1 = 0 \\ y'(0) &= C_0 = 1 \end{aligned}\right. \Rightarrow y(t) = \sin t$$

ex) $y'' + y = 0$ for $0 \le t \le 1$

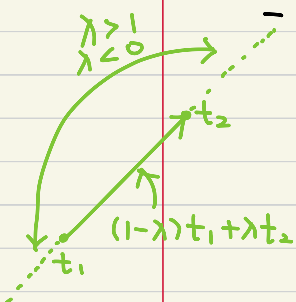
$y(0) = 1$, $y'(0) = 0$

$\Rightarrow$ $y(t) = \cos t$

Sec 5.1. The elementary theory of IVPs

- Lipschitz condition (function)

$$|f(t, y_1) - f(t, y_2)| \le L\,|y_1 - y_2|$$



- Convex (domain)

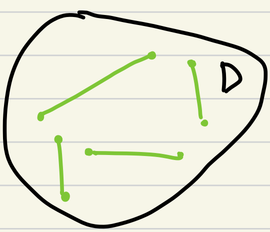
(t_1, y_1) , (t_2, y_2) : two points in D.

$((1-\lambda)t_1 + \lambda t_2 , (1-\lambda)y_1 + \lambda y_2)$, $0 \le \lambda \le 1$

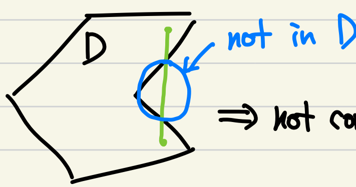
: a line segment between two points.

every line segment is in D.

$\Rightarrow$ D is convex.



$\Rightarrow$ convex



$\Rightarrow$ not convex.

Thm) $f(t, y)$: a function,
D : a convex domain.
If $\left|\frac{\partial f}{\partial y}(t, y)\right| \le L$, for all $(t, y) \in D$,
then f satisfies a Lipschitz condition.

MVT : $f(y_2) - f(y_1) = f'(\xi)(y_2 - y_1)$
$y_1 \le \xi \le y_2$

$$\left| f(t, y_2) - f(t, y_1) \right| = \left| \frac{\partial f}{\partial y}(t, \xi) \right| \left| y_2 - y_1 \right|$$
$$\le L \left| y_2 - y_1 \right|$$

$y_1 \le \xi \le y_2$
$\Rightarrow$ $\cancel{(t, y_1) \le (t, \xi) \le (t, y_2)}$