04/16/20

- RK 4

Compare

- Taylor method (order 4)

$$w_{i+1} = w_i + h T^{(4)}(t_i, w_i)$$

$$T^{(4)}(t, y) = f(t, y) + \frac{h}{2} f'(t, y) + \cdots + \frac{h^3}{4!} f^{(3)}(t, y)$$

$$f'(t,y) = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial y} f$$

- RK 4 method

$$w_0 = \alpha,$$

$$w_{i+1} = w_i + \frac{h}{6} f(t_i, w_i) + \frac{h}{3} \underline{f(t_i + \alpha_1 h, w_i + \delta_1 h f(t_i, w_i))}$$

$= f(t_i, w_i) + \cdots$

$$+ \frac{h}{3} \underline{f(t_i + \alpha_2 h, w_i + \delta_2 h f(t_i + \gamma_2 h, w_i + \gamma_3 h f(t_i, w_i)))}$$

$= f(t_i, w_i) + \cdots$

$$+ \frac{h}{6} \underline{f(t_i + \alpha_3 h, w_i + \delta_3 h f(t_i + \gamma_4 h, w_i + \gamma_5 h f(t_i + \gamma_6 h, w_i + \gamma_7 h f(t_i, w_i))))}.$$

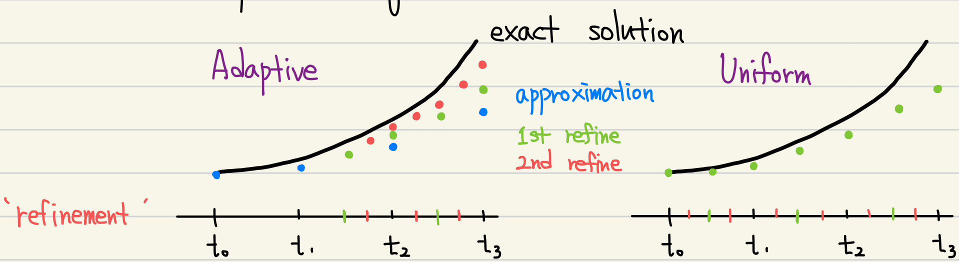
$= f(t_i, w_i) + \cdots$

$$\hookrightarrow \frac{1}{6} f(t_i, w_i) + \frac{1}{3} f(t_i, w_i) + \frac{1}{3} f(t_i, w_i) + \frac{1}{6} f(t_i, w_i) = f(t_i, w_i)$$

Sec 5.5. Error Control and the Runge–Kutta–Fehlberg Method (RKF)

$$\begin{cases} w_0 = \alpha \\ w_{i+1} = w_i + h\,\phi(t_i, w_i, h) \end{cases}$$

– Adaptive algorithm



1st :	6 points	7 points
2nd :	9 points	13 points
3rd :	big difference (# of pts)	

'We don't know the exact solution, so we don't compute the error.'

- Which intervals needs to be refined without the exact solution?
- How much do we need to refine?

- Error control (indicator for refinement)

higher-order method ex) RK2 : w_{i+1}

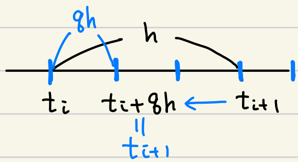
RK3 : $\tilde{w}_{i+1}$

$$\frac{q^n}{h}\left|\tilde{w}_{i+1} - w_{i+1}\right| \le \varepsilon$$

$\approx |T_{i+1}(qh)|$

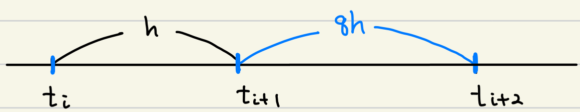
because higher-order method gives more accurate approximations than w_{i+1} so that we can consider $\tilde{w}_{i+1}$ as the exact y_{i+1}.

$\Rightarrow$ $q < 1$ $\rightarrow$ reduced step size (refine)



$(qh < h)$

$q \ge 1$ $\rightarrow$ not change (not refined)



$(qh \ge h)$

Quiz 2.

c. $y' = 1 + y/t, \quad 1 \le t \le 2, \quad y(1) = 2,$ with $h = 0.25$; actual solution $y(t) = t\ln t + 2t$.

1) Mid-point 2) RK 3