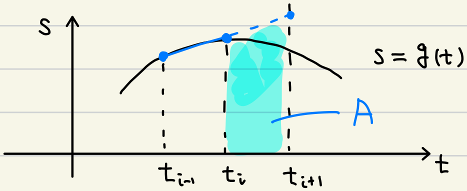
04/23/20 ex) Adams - Bashforth two - step explicit method

$(t_{i-1}, g(t_{i-1}))$
$(t_i, g(t_i))$



$$\int_{t_i}^{t_{i+1}} g(t)\,dt \approx A$$

$$P_1(t) = at + b$$
$$= \frac{g(t_i) - g(t_{i-1})}{t_i - t_{i-1}} t + \frac{g(t_{i-1})t_i - g(t_i)t_{i-1}}{t_i - t_{i-1}}$$

$$A = \int_{t_i}^{t_{i+1}} P_1(t)\,dt$$
$$= \int_{t_i}^{t_{i+1}} \frac{g(t_i) - g(t_{i-1})}{t_i - t_{i-1}} t + \frac{g(t_{i-1})t_i - g(t_i)t_{i-1}}{t_i - t_{i-1}}\,dt$$

$$\int_{t_i}^{t_{i+1}} t\,dt = \frac{t_{i+1}^2 - t_i^2}{2}$$

$$= \frac{g(t_i) - g(t_{i-1})}{t_i - t_{i-1}} \left(\frac{1}{2}(t_{i+1}^2 - t_i^2)\right)$$
$$+ \frac{g(t_{i-1})t_i - g(t_i)t_{i-1}}{t_i - t_{i-1}} (t_{i+1} - t_i)$$

'$h = t_i - t_{i-1} = t_{i+1} - t_i$'

$$= \frac{1}{2}(g(t_i) - g(t_{i-1}))(t_{i+1} + t_i)$$
$$+ g(t_{i-1})t_i - g(t_i)t_{i-1}$$
$$= \frac{1}{2} g(t_i)(t_{i+1} + t_i - 2t_{i-1})$$
$$+ \frac{1}{2} g(t_{i-1})(2t_i - t_{i+1} - t_i)$$

$$t_{i+1} + t_i - 2t_{i-1}$$
$$= t_{i+1} - t_{i-1} + t_i - t_{i-1}$$
$$= 2h + h = 3h$$
$$2t_i - t_{i+1} - t_i$$
$$= t_i - t_{i+1} = -h$$

$$= \frac{1}{2} g(t_i)(3h) - \frac{1}{2} g(t_{i-1}) h$$

Sec 5.9. High-Order Equations and Systems of Differential Equations.

- System of DEs (two equations)

$$\begin{cases} u_1' = f_1(t, u_1, u_2) \\ u_2' = f_2(t, u_1, u_2) \end{cases} \qquad (\vec{u}' = \vec{f}(t, \vec{u}))$$

  - Initial condition

    $u_1(a) = \alpha_1 \quad , \quad u_2(a) = \alpha_2$

  - Euler's method

    $\vec{w}_{i+1} = \vec{w}_i + \vec{f}(t_i, \vec{w}_i)$

    $$\Rightarrow \quad \begin{array}{l} w_{i+1}^1 = w_i^1 + f_1(t_i, w_i^1, w_i^2) \\ w_{i+1}^2 = w_i^2 + f_2(t_i, w_i^1, w_i^2) \end{array}$$

- High-order equations (second-order ODEs)

$$u'' = f(t, u, u')$$

  - Initial condition

    $u(a) = \alpha \quad , \quad u'(a) = \beta$

  - Convert high-order DEs to the system.

    $u_1' = u_2$

    $u_2' = f(t, u_1, u_2)$,

    with initial conditions $u_1(a) = \alpha$, $u_2(a) = \beta$

Quiz 3.

1. Derive Simpson's method by applying Simpson's rule to the integral

$$y(t_{i+1}) - y(t_{i-1}) = \int_{t_{i-1}}^{t_{i+1}} f(t, y(t))\, dt.$$

2. Simpson's method is an (explicit / implicit) and (two-step / three-step) method. Choose your answer and explain why.